Plane covering by mobile sensors *

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Аннотация

In the well studied regular covers of a plane region by disks, the region is tiled by the equal regular polygons (tiles), and all tiles are covered equally. In this case the centers of disks are placed in the certain points of each tile. In the sensor network the disk is a sensing area of a sensor. But sometimes it is impossible to place the sensors in the certain points, and they are distributed randomly over the region. In the last case it is not clear how to organize the network and how to perform analysis of its efficiency. We proposed the methods to build the "pseudoregular" covers on the different grids and to gain the sensor's mobility. We compare the lifetimes of mobile wireless sensor networks and static network in case of equal distribution of sensors over the region.

Introduction

Wireless sensor network (WSN) is presented by the set J, $|J| = m$, of mobile sensors with adjustable sensing and communication ranges, which are distributed over the plane region O of space S. Each sensor, being in active mode, consumes its limited energy for sensing, communication and movement. In a sleep mode sensor preserves its energy. Let the monitoring and communication areas of every sensor are the disks of certain radii with sensor in the centers [1, 2, 4, 5]. The region O is covered if every its point belongs to at least one monitoring disk. The arc (i, j) belongs to the communication

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network, if sensor j is inside the communication range of sensor i. A lifetime of WSN is the number of time rounds during which the region O is covered by the connected active sensors [3]. The problem is to maximize the lifetime of WSN. This problem is very complex, and even special cases remain NP-hard [6]. Our goal is to take an advantage of mobility of sensors in comparison with the static sensor model in class of regular covers [4, 5].

Suppose that each sensor has the energy storage $q > 0$. For any sensor, a sensing energy consumption per time round depends on a sensing range r (radius of the disk) and equals $SE = \mu_1 r^a$, $\mu_1 > 0$, $a \geq 2$; a communication energy consumption per time round depends on the distance d and equals $CE = \mu_2 d^b$, $\mu_2 > 0$, $b \geq 2$; and the energy consumption per time round during the motion depends on the speed v and equals $ME = \mu_3 v^c$, $\mu_3 > 0$, $c > 0$. We suppose that during the motion sensor do not consumes the energy for sensing and communication.

Let some regular grid, which tiles the region O, is given. In a regular cover the sensors, as usually, placed in the grid nodes and cover some disk area (if grid node has number i, then we call the disk it covers as *disk* i). Let sensors are distributed *uniformly* over the region O, and parameter $a_{ij} = 1$ if i is the closest grid node to the sensor j and $a_{ij} = 0$ otherwise. Denote the set $J_i = \{j \in J | a_{ij} = 1\}$, and we reasonably suppose that the sensor j in J_i, if it is active, must cover the disk i. Then if sensor j is located on a distance r away from the grid node i, then it must increase the sensing range by r in order to cover the disk i. Moreover, if the distance between the node i and sensor $j_1 \in J_i$ is r_1, and the distance between the node k and sensor $j_2 \in J_k$ is r_2, then in order to guarantee a communication between the sensors j_1 and j_2 it is necessary to increase the communication ranges of j_1 and j_2 by at least $r_1 + r_2$ units. Since the sensors are mobile, then every sensor $j \in J_i$ can move towards the node i during some time rounds in order to be nearer to i. For the sake of simplicity, we suppose that the speed of every sensor is 0 or v. Therefore, if sensor $j \in J_i$ moves, then the speed is v and the direction is towards the grid node i.

In [7] we've presented an approach which permits to compare static and mobile regular covers when the grid is triangular. In this paper we continue research for rectangular grid and compare the new results with the results we got in [7]. In the next section we briefly remind the results in [7].

1 Triangular regular grid

In [7] we considered two cases: when the grid is fixed and when the grid is transposed. We call the last one a free grid.

In the first case region O is tiled by the regular triangles (tiles) of the side $R\sqrt{3}$. These triangles form a regular grid with the set of grid nodes – nodes of triangles. If the sensors are distributed uniformly, then we show in [7] that the lifetime of mobile WSN equals

$$\Lambda_T^\delta \approx \sum_{k=1}^{\lfloor \frac{R\sqrt{3}}{2v} \rfloor} \frac{(q - l_k\mu_3 v^c)N_k}{\mu_1(R + (k - l_k)v)^a + \mu_2(R\sqrt{3} + 2(k - l_k)v)^b}, \tag{1}$$

where $N_k \approx m\pi(2k - 1)v^2/S$, $K = \lfloor \delta/v \rfloor$ ($\lfloor A \rfloor$ is the integer part of A), $\delta = R\sqrt{3}/2$, $l_1 = 0$ and

$$l_k = \arg\max_{0\le l\le k} \frac{q - l\mu_3 v^c}{\mu_1(R + (k - l)v)^a + \mu_2(R\sqrt{3} + 2(k - l)v)^b}, \quad k \ge 2.$$

In free grid we suppose that the number of sensors N_k in $J_i^k = \{j \in J_i | \delta_{k-1} < d_{ij} \le \delta_k\}$, where $\delta_k = k \cdot v$ and d_{ij} is the distance between the sensor j and grid node i, is sufficiently great for each $k = 1, \ldots, K$. If the grid is wandering, then we proved in [7] that the lifetime is this case is

$$\Lambda_T^v \ge \left(\frac{3R^2 m\pi}{4S} - \frac{\mu_1(R + v)^a + \mu_2(r\sqrt{3} + 2v)^b}{q} \right) \frac{q - \mu_3 v^c}{\mu_1 R^a + \mu_2(R\sqrt{3})^b} +$$
$$\frac{6mR^2}{25S} \frac{q - 2\mu_3 v^c}{\mu_1 R^a + \mu_2(R\sqrt{3})^b}$$

2 Rectangular regular grid

Let's consider again two cases: when the grid is fixed and when the grid is transposed. Let the region O is tiled now by equal squares of the side $R\sqrt{2}$. These squares form a regular grid. If all sensors have the same sensing range R and they are equally placed in the grid nodes, then this cover we call $R1$. In the cover $R1$ each quadruple of neighbor disks of radius R with centers in the nodes of square has one common to these four disks point in the center of the tile. In cover $R1$ each active sensor, located in the node i, must cover the disk of radius R centered in the node i (*disk* i). Then the sensing energy consumption of every sensor equals $SE = \mu_1 R^a$. The communication distance for each sensor in cover $R1$ is $R\sqrt{2}$, hence the communication energy consumption per time round is $CE = \mu_2(R\sqrt{2})^b$, and then the lifetime of any sensor is $t_{R1} = q/(\mu_1 R^a + \mu_2(R\sqrt{2})^b$. Since the number of grid nodes in $R1$ is $N_{R1} \approx S/(2R^2)$, then the lifetime of regular cover $R1$ equals

$$L_{R1} \approx \frac{t_{R1}m}{N_{R1}} \approx \frac{2qm}{S(\mu_1 R^{a-2} + \mu_2 R^{b-2}(\sqrt{2})^b)}.$$

If sensors are distributed uniformly over the O, then the sensors inside the regular *square* i with center in the node i and the sides at the distance $\delta = R/\sqrt{2}$ from the center, are in the set J_i. Let's consider the concentric circles of radii $\delta_k = k \cdot v$, $k = 1, \ldots, K$, with centers in some grid node. Any sensor $j \in J_i^k = \{j \in J_i | \delta_{k-1} < d_{ij} \leq \delta_k\}$ could reach the grid node i by at most k time rounds.

Since the resource of each sensor is limited by q, then if any sensor $j \in J_i^k$ moves l time rounds and, as a result, consumes its energy, then, taking into account the remainder sensor-node distance $(k-l)v$, it can be active during $t_k(l) = (q - l\mu_3 v^c)/(\mu_1(R + (k-l)v)^a + \mu_2(R\sqrt{2} + 2(k-l)v)^b)$ time rounds. Function $t_k(l)$ is concave, then one can find $l_k = \arg\max\limits_{0 \leq l \leq k} t_k(l)$ by $O(\log_2 K)$ time complexity. Since the sensors are distributed uniformly, then there are $N_k \approx m\pi(2k-1)v^2/S$ sensors in every set J_i^k. Let first active sensors are initially located in J_i^1, and we suppose that they do not move and are active during $L_1 \approx qN_1/(\mu_1(R+v)^a + \mu_2(R\sqrt{2} + 2v)^b)$ time rounds. During time L_1 sensors in J_i^2 could move towards the grid node i, and then they can be active during $L_2 = N_2 \max\limits_{0 \leq l \leq \min\{2, L_1\}} t_2(l)$ time periods. Therefore, during the time $\Lambda_{k-1} = \sum\limits_{l=1}^{k-1} L_l$ sensors in J_i^k could move to the grid node i, and then they can be active during $L_k = N_k \max\limits_{0 \leq l \leq \min\{k, \Lambda_{k-1}\}} t_k(l)$ time periods. Setting $l_1 = 0$, the lifetime of such mobile WSN equals

$$\Lambda_R^\delta \approx \sum_{k=1}^{\lfloor \frac{R}{v\sqrt{2}} \rfloor} \frac{(q - l_k \mu_3 v^c) N_k}{\mu_1(R + (k - l_k)v)^a + \mu_2(R\sqrt{2} + 2(k - l_k)v)^b}. \tag{2}$$

Let's compare (1) and (2). The k-th summand in (1) is always less than the k-th summand in (2). But the number of summands in (1) is greater than in (2). Then Λ_T^δ can be both more than Λ_R^δ and vice versa. For example, if $l_k = k$, $v = 1$, $R = 8$, $\mu_1 = 5$, $\mu_2 = 1$, $\mu_3 = 10$, $a = b = 2$, then

$$\Lambda_T^\delta \approx \frac{qN_1}{656,43} + \sum_{k=2}^{6} \frac{(q - 10k)N_k}{512} \approx \frac{m\pi}{448S}(713q - 14000)$$

and

$$\Lambda_R^\delta \approx \frac{qN_1}{582,25} + \sum_{k=2}^{5} \frac{(q - 10k)N_k}{448} \approx \frac{m\pi}{448S}(25q - 9400).$$

Therefore, if $q < 0,6686$, then $\Lambda_R^\delta < \Lambda_T^\delta$, else $\Lambda_R^\delta \geq \Lambda_T^\delta$.

In the free greed we suppose that the number of sensors N_k in J_i^k is sufficiently great for each $k = 1, \ldots, K$. If grid is wandering, then we may displace it several times without change the size (the new grid node i_n is relocated 2δ away from the previous position i_{n-1} to the right and down), then WSN's lifetime can be increased as follows. Let us set $\delta = v$ and suppose that during the first time round, when a part of sensors in $J^1_{i_1}$ are active, other sensors in every set $J^1_{i_n}$, $n \geq 1$, move to the grid node i_n. The number of sensors in each set $J^1_{i_1}$, which are active during the one (first) time round, equals $n'_1 \approx (\mu_1(R+v)^a + \mu_2(R\sqrt{2}+2v)^b)/q$, and we suppose that $n'_1 \leq N_1$. These sensors don't move and the active ones must increase their sensing ranges by v. During the first time round $N_1 - n'_1$ sensors in each set $J^1_{i_1}$ can reach the grid node i_1, and it is not necessary to increase their sensing ranges to cover the O. Moreover, each sensor in $J^1_{i_n}$, $n \geq 2$, can reach i_n during the first time round. The number of sensors in every set $J^1_{i_n}$, $n \geq 2$, is N_1, and the number of these sets (new grid nodes) is $n'_2 \geq \lfloor R\sqrt{2}/(2v)\rfloor^2 - 1$ ($n'_2 + 1$ is the number of disks of radius δ packed in the square of side $R\sqrt{2}$). Every sensor, located outside the sets $J^1_{i_n}$, $n \geq 1$, has two time rounds to reach the nearest grid node. The number of such sensors in the square is $n'_3 \approx 2R^2m/S - (n'_2+1)N_1 \geq 2R^2m/S - R^2N_1/(2v^2)$. Since $N_1 \approx m\pi v^2/S$, then the WSN's lifetime in this case is

$$\Lambda^v_R \approx 1 + (N_1 - n'_1 + N_1 n'_2)\frac{q - \mu_3 v^c}{\mu_1 R^a + \mu_2(R\sqrt{2})^b} + n'_3\frac{q - 2\mu_3 v^c}{\mu_1 R^a + \mu_2(R\sqrt{2})^b}.$$

If one suppose inequality $(\mu_1(R+v)^a + \mu_2(R\sqrt{2}+2v)^b)/q$, i.e. the resource of every sensor is big enough, then

$$\Lambda^v_R \approx \frac{m}{2S}\frac{2v^2\pi q - 2v^2\pi\mu_3 v^c + 4qR^2 - 8\mu_3 v^c R^2 + \mu_3 v^c \pi R^2}{\mu_1 R^a + \mu_2(R\sqrt{2})^b}.$$

For triangular grid under the similar assumptions, the lifetime is

$$\Lambda^v_T \approx \frac{m}{100S}\frac{75R^2\pi(q - \mu_3 v^c) + 24R^2(q - 2\mu_3 v^c)}{\mu_1 R^a + \mu_2(R\sqrt{3})^b}.$$

Set, for example, $v = 1$, $R = 8$, $\mu_1 = 5$, $\mu_2 = 1$, $\mu_3 = 10$, $a = b = 2$ and let's compare Λ^v_T and Λ^v_R. In this case $m\Lambda^1_T/S = F^1_T \approx (52q - 267)/160$, $m\Lambda^1_R/S = F^1_R \approx (131q - 1587)/448$, and F^1_T is always greater than F^1_R. If $R = 2$, and the other parameters conserve their values, then $F^1_T < F^1_R$ if $q > 13,48$, and $F^1_T \geq F^1_R$ otherwise.

3 Conclusion

The mobility of sensors is the unquestionable advantage. But this additional option must be used optimally. We consider the triangular and rectangular regular covers of a plane area by equal disks and proposed two ways to use sensor's mobility to construct a pseudoreqular covers. We evaluate these covers by estimating the low bound for WSN's lifetime and comparing with static networks. In [7] we've done a part of work for triangular pseudoreqular covers. In this paper we continue our research for the rectangular pseudoreqular covers. We show that any pseudoreqular cover could be preferable. It depends on the parameters values.

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