

APPLICATION OF NONLINEAR ADAPTATION ON MANIFOLDS FOR ECONOMIC OBJECTS

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An application of the method of analytical design of aggregated regulators for constructing systems of control over integrated economic entities with positive and negative feedback loops is discussed. The controlled objects are presented as a system of ordinary non-linear differential or difference equations with chaotic behavior in the case of certain combinations of parameters. A number of control problem formulations are given and the algorithms of control synthesis are reported and theoretically validated. Some illustrative examples of application of the proposed algorithms are provided along with their numerical simulation data. The results obtained would be useful in designing a smart control system and for real-time decision making in the financial policies of a variety of economic entities.

Keywords: non-linear multi-dimensional object, disturbance compensation, deterministic chaos, analytical design of aggregated regulators.

Introduction

The analysis and control approaches implementing the principle of directed self-organization and decomposition of non-linear dynamic systems is currently becoming popular in the theory of non-linear systems [1]– [7].

The attractiveness of such methods of analytical design of aggregated regulators (ADAR) proposed by the Kolesnikov school of sciences [3] as backstepping [7] and sliding-mode control [5] consists in inclusion of the physical features of a controlled object into consideration [4] in the course of synthesis, which is done via specially selected invariants forming target manifolds or desired goal attractors of a nonlinear system.

Nonlinear models of economic entities of different scales and business application areas (production, distribution, retail and/or wholesale businesses, banking institutions, etc.) are becoming increasingly more appealing to the researchers, as they approach real physical models and are described by multivariable nonlinear indices responsive to the positive and negative feedback loops.

This paper deals with an application of the techniques of nonlinear adaptation on manifolds [3, 8] for economic models; in particular, based on a unified methodology we formulate and solve a number of different control problems for the following types of objects with nonlinear models and unstable behavior: capital expansion of a corporate giant, changes in sales volumes of two competing companies producing two similar commodities, and financial operations of a small business enterprise. All of these objects are represented by systems of difference equations or ordinary nonlinear differential equations with chaotic behavior in the cases of certain parameter combinations (models of deterministic chaos). In the open-loop states, the mathematical models of economic objects are thought to be unstable, which in essence means the following: there are such initial conditions and parameter values inherent in these objects that the property of economic self-regulation is no longer valid for them [9].

The purpose of this study is to design a programmed control system for the model of a small business enterprise derived from the Lorentz model [11], which in turn accounts for a number of processes, including temporal variations in economic performance indicators and ecological indices.

For the sake of illustration, we consider here the operation and performance of a small-scale IT service company (providing services of software engineers and programmers in the field of new application design, specialized software development, etc.).

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1 Problem Statement

1.1 Mathematical Model of Object

Let us look at a controlled object described by the following system of equations (1):

$$\begin{aligned}\dot{x}_1(t) &= \alpha x_2 x_1 - \gamma x_1 + z_1 + u_1, \\ \dot{x}_2(t) &= \mu(x_1 + x_2) - \beta x_1 x_3, \\ \dot{x}_3(t) &= \delta x_2 - \lambda x_3 + z_2 + u_2,\end{aligned}\tag{1}$$

where $x \in R^3, u \in R^2, z \in R^2$ are the vector variables of control and unknown disturbance, respectively.

It is required to carry out control in the state space of the object transferring this object from its given initial state $x(0) = (x_1(0), x_2(0))$ into the neighborhood of the target manifold $\psi(x(t)) = (\psi_1(x(t)), \psi_2(x(t)))$, and minimizing the quality functional J , taking into account two-dimensional control variable

$$J = \sum_{i=0}^2 \int_0^\infty ((\Psi_i(x(t)))^2 + (\omega_i)^2 (\dot{\Psi}_i(x(t)))^2) dt \rightarrow \min\tag{2}$$

In order to solve the stabilization problem for variables x_1, x_2 , macrovariables $\psi_1(x), \psi_2(x)$ can be, for instance, selected as follows:

$$\psi_1(x(t)) = x_1(t) - x_{10}, \psi_2(x(t)) = x_2(t) - x_{20}\tag{3}$$

where quantities x_{10}, x_{20} are the target values of variables x_1, x_2 .

1.2 Basic Requirements to the Control Object

The control synthesis algorithm with a program-defined adaptation to a given manifold assumes that the following standard requirements be fulfilled: there is an asymptotically stable target system as a whole satisfying the desired technological tasks; all solutions of the initial system have to be limited; the initial model of the object's state has to be stabilizable; there is an attractive target manifold described by $\psi(x) = 0$ with respect to the object's initial system of equations, where $\psi(x)$ is a certain continuous differentiable vector function meeting the desired physical properties of the target state, which is referred to as a goal macro-variable, whose dimension coincides with that of the control vector, and the quality functional of the synthesized control system is given by (2).

Weighting factors $\omega_i, i = 1, 2, 3$ are the parameters of the regulator settings; they are proportional to the duration of motion of the image point to the target manifold (target attractor) $\psi(x) = 0$. Given these assumptions, the mathematical formulations of all problems listed above would be transformed into the following: it is necessary to maintain control in the space of states of an object transferring this object from its predetermined initial state $x(0)$ into the neighborhood of the target manifold $\psi(x) = 0$ and minimizing the quality functional J , taking into account 2-dimension of the control variable.

Remark. Functional (2) has a clear physical meaning: it generalizes the classical quadratic functional. In particular, the first term in the expression for functional J represents different functional properties of the controlled object, and the second term the property of the synthesized control system for a dynamic object, specifically, the rate of its approach towards the stable state.

2 Solution of the control problem

Lemma. Let there exist a variational problem $J, \psi(x) = 0$ where

$$J = \int_0^\infty F(t, \psi, \dot{\psi}) dt \rightarrow \min, F(t, \psi, \dot{\psi}) = (\phi(\Psi(x(t))))^2 + (\omega)^2 (\dot{\Psi}(x(t)))^2,\tag{4}$$

with a restriction $\psi(x) = 0$.

If $\phi(\Psi)$ is single-valued, continuous and differentiable function for all Ψ such that $\phi(0) = 0, \phi(\Psi)\Psi > 0 \forall \Psi \neq 0$, the Euler-Lagrange equation for the problem (1), (2) would be given by $\omega \ddot{\Psi} + \phi(\Psi) = 0$.

The proof of this lemma relies directly on the results of theoretical mechanics and follows from [10] e.g. This lemma plays the main role in finding control with predetermined target properties of an object.

2.1 Algorithm of synthesis of vector control for the object (1), (2)

1. Extend the phase space to transform the initial open-loop system into a closed-loop one

$$\begin{aligned}
 \dot{x}_1(t) &= \alpha x_2 x_1 - \gamma x_1 + z_1 + u_1, \\
 \dot{x}_2(t) &= \mu(x_1 + x_2 x_2) - \beta x_1 x_3, \\
 \dot{x}_3(t) &= \delta x_2 - \lambda x_3 + z_2 + u_2, \\
 \dot{z}_1(t) &= \eta_1 \Psi_1, \\
 \dot{z}_2(t) &= \eta_2 \Psi_2,
 \end{aligned} \tag{5}$$

where $\eta_1 > 0, \eta_2 > 0$.

The form of the right-hand parts of the last two equations of the system (5) is validated by the Lyapunov's second theorem.

2. Determine the first auxiliary vector macro-variable $\Psi^{(1)} = (\Psi_1^{(1)}, \Psi_2^{(1)})$:

$$\begin{aligned}
 \Psi_1^{(1)} &= \Psi_1 + k_1 z_1, k_1 > 0, \\
 \Psi_2^{(1)} &= \Psi_2 - \varphi(x_2, z_1, z_2),
 \end{aligned}$$

accurate within the unknown in this step of function $\varphi()$.

3. Formulate a variational problem given by $(J_1, \psi^{(1)})$ as follows:

$$J_1 = \sum_{i=0}^2 \int_0^\infty ((\Psi_i^{(1)}(x(t)))^2 + (\omega_i^{(1)})^2 (\dot{\Psi}_i^{(1)}(x(t)))^2) dt \rightarrow \min \tag{6}$$

with a restriction $\psi^{(1)}(x) = 0$.

A solution of $(J_1, \psi^{(1)})$ yields a system of equations for vector control $u = (u_1, u_2)$ (in accordance with lemma):

$$\omega_i^{(1)} \dot{\Psi}_i^{(1)} + \Psi_i^{(1)} = 0, i = 1, 2. \tag{7}$$

4. Solve the system of functional equations (7) for variables u_1, u_2 :

$$\begin{aligned}
 u_1 &= (\gamma - (\omega_1^{(1)})^{-1})x_1 - \rho_1(\mu + (\omega_1^{(1)})^{-1})x_2 - \rho_1\mu x_3 + \beta\rho_1 x_1 x_3 - \alpha x_2 x_3, \\
 u_2 &= (\lambda - \rho_2\mu - (\omega_2^{(1)})^{-1})x_3 - (\delta + \rho_2\mu + (\omega_2^{(1)})^{-1}\rho_2)x_2 + \beta\rho_2 x_1 x_3.
 \end{aligned}$$

The forms of control u_1, u_2 ensure a transfer of the object (1) into the neighborhood of the target manifold $\Psi^{(1)} = 0$, on which the following relations are fulfilled:

$$\Psi_1^{(1)} = 0 \rightarrow \Psi_1 = -k_1 z_1, \Psi_2^{(1)} = 0 \rightarrow x_3 = \varphi. \tag{8}$$

The first relation (8) guarantees an asymptotic stability to the solutions of the fourth equation of the system (5) due to the fact that the following equality holds true $\dot{z}_1 = -k_1 \eta_1 z_1$.

Thus within this step of the algorithm, one of the target invariants $\Psi_1 = 0$ is obtained.

5. Decompose the initial system of equations (5) on manifold (8). In the neighborhood of manifold $\Psi^{(1)} = 0$ motion of the image point of the system is described by the following system of differential equations:

$$\begin{aligned}
 \dot{x}_2(t) &= \mu(x_2 + \varphi) - \beta(x_{10} - k_1 z_1)\varphi, \\
 \dot{z}_2(t) &= \eta_2 \Psi_2,
 \end{aligned} \tag{9}$$

6. Assume the second scalar macro-variable $\Psi^{(2)} = \Psi_2 + k_2 z_1, k_2 > 0$ in order to define function φ from (9).

7. Pose the second variational problem given by $(J_2, \psi^{(2)})$ where

$$J_2 = \int_0^\infty ((\Psi^{(2)}(x(t)))^2 + (\omega^{(2)})^2 (\dot{\Psi}^{(2)}(x(t)))^2) dt \rightarrow \min \quad (10)$$

with a restriction $\psi^{(2)}(x) = 0$. Solving the problem $(J_2, \psi^{(2)})$ yields an equation for the sought-for-function φ (in accordance with the lemma):

$$\omega^{(2)} \dot{\Psi}^{(2)} + \Psi^{(2)} = 0, \quad (11)$$

8. Solve the functional equation (11) for variable φ :

$$\varphi(x_2, z_1, z_2) = ((-\mu x_2 - k_2 \eta_2 (x_2 - x_{20}) - (\omega^{(2)})^{-1} (\Psi_2 + k_2 z_2)) (\mu - \beta (x_{10} - k_1 z_1))^{-1}, \quad (12)$$

3 Results of numerical simulation

The plots of controlled coordinates presented in Fig. 1 d), e) suggest an acceptable quality of control (Fig. 1 c)) over the object, which ensures that the goal is achieved $\psi = 0$ (in Fig. 1 f)) in terms of the global minimum of functional J and the control object is sustained in the neighborhood of the desired manifold $\psi = 0$.

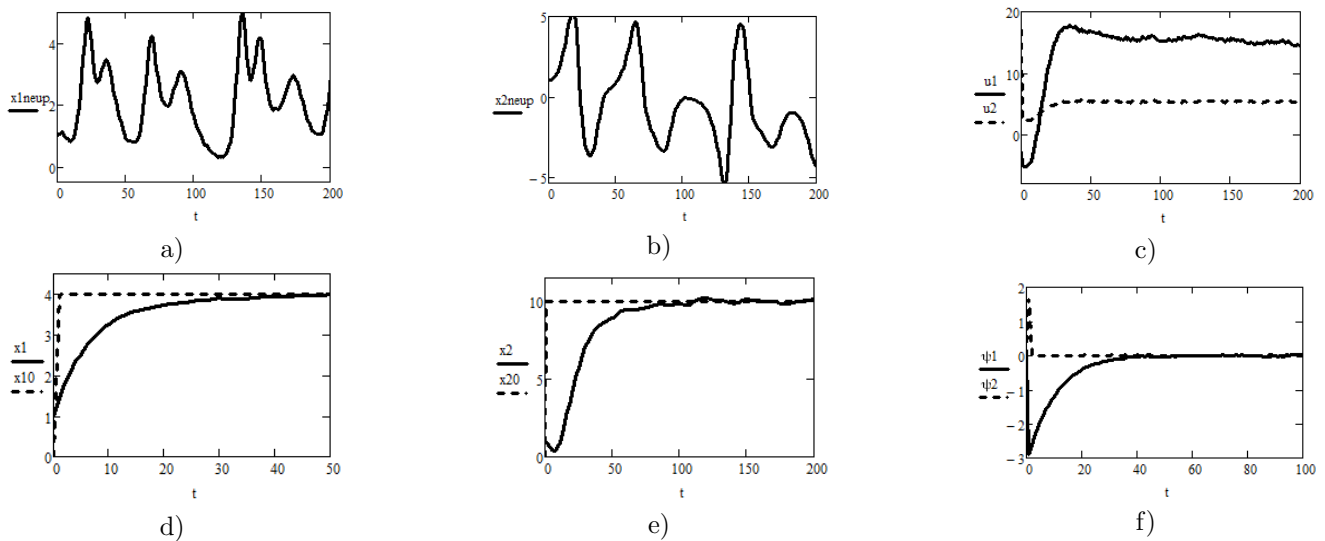


Figure 1: Results of control system simulation: a) plot of uncontrolled coordinate x_1 , b) plot of uncontrolled coordinate x_2 , c) plot of control variables u_1, u_2 , d) plot of controlled coordinate x_1 , e) plot of controlled coordinate x_2 , f) plot of target macro-variables ψ_1, ψ_2 .

4 Conclusion

In this study we have constructed two control systems for a dynamic model of operations of a small business enterprise, relying on the analytical design of aggregated regulators representing different methods of organizing control over an economic object. The resulting systems of control have been tested using initial data obtained from the performance indicators of a real small business enterprise.

The results obtained can be useful for solving applied economic problems in decision-making on re-distribution of the flow of funds in order to ensure stable operations of small businesses, since the proposed control law represents a procedure for a feedback with a pre-emptive impact.

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