

# ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ГРАВИТАЦИОННЫХ ПОТОКОВ В СТРАТИФИЦИРОВАННОЙ АТМОСФЕРЕ С ИНВЕРСИЕЙ

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В работе исследуется влияние стратификации и инверсии на приземное давление при распространении гравитационного течения (атмосферного холодного фронта) над плоской орографией с помощью конечно-разностной негидростатической модели динамики атмосферы. С целью сравнения с имеющимися в литературе данными моделирования, физический процесс считается невязким и адиабатическим, исключено также влияние орографии. В модели используется система уравнений Навье — Стокса в приближении Буссинеска. Эффективная численная реализация системы уравнений достигается введением искусственной сжимаемости. Поверхность фронта описывается в модели специальным уравнением. Приводятся результаты расчетов приземного давления для нейтральной и устойчивой стратификации при наличии в атмосфере слоя инверсии. При устойчивой стратификации, по сравнению с нейтральной, фронт движется быстрее и имеется резкий скачок давления гораздо раньше, чем фронт доходит до точки наблюдения. Это явление описывается современной теорией атмосферных фронтов. Результаты расчетов находятся также в качественном согласии с данными моделирования по конечно-разностной модели с исходной генерацией фронта протяженным источником холодного воздуха.

**Ключевые слова:** гравитационный поток, стратификация, инверсионный слой.

## 1 Introduction

A front in the atmosphere is a phenomenon of gravitational flows that take place in variety of forms: breeze fronts, storm flows, squall-lines, etc [1]. Phenomena of great importance are cold atmospheric fronts propagating near the surface with high speeds. These fronts may be retarded and changed in shape under the influence of the underlying surface and stratification of the atmosphere. A change in the shape of a frontal system under the influence of inversion layers is a commonly observed phenomenon[2, 3]. Atmospheric gravity currents occupy a wide range of length scales from several meters to thousands of kilometers. These currents may be subdivided into classes varying from micro- to macro-scales. Mesoscale flows are defined to lie approximately in the interval from two to two thousand kilometers[4]. The flows of interest in the present study are mesoscale currents. These flows are relatively shallow: they belong to the atmospheric boundary layer and range in an interval of only a few kilometers from the surface, in the lower atmosphere. An important paper in the literature on theoretical and numerical studies of atmospheric fronts is [5]. Two distinct approaches can be recognized in the numerical simulation of front propagation. In one approach, the front to be calculated is considered to be a gravity current driven by a cold air source [3]. In the other, the front surface is considered to be a passive scalar, a tracer, to distinguish between warm and cold air masses [6]. The purpose of the present paper is to estimate the stratification and inversion effects on surface pressure in the propagation of an atmospheric gravity current (cold front) over flat terrain. The study is carried out with a non-hydrostatic finite-difference model of atmospheric dynamics. Artificial compressibility is introduced into the model in order to make its equations hyperbolic, which greatly simplifies the formulation of the boundary conditions and the numerical treatment of the model[7]. For comparison with the available simulation data, the physical processes under study are assumed to be non-viscous and adiabatic. The influence of orography is also eliminated. The results of simulations of surface pressure under neutral stratification and stable stratification with an inversion layer are presented.

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## 2 Model equations

The Navier — Stokes equations for a compressible air flow are used here for the calculation of gravity flows in a stratified atmosphere. In a three-dimensional statement, the basic equations of motion, heat, moisture and continuity, written in a terrain-following coordinate system, are as follows:

$$\frac{dU}{dt} + \frac{\partial P}{\partial x} + \frac{\partial(G^{13}P)}{\partial \eta} = f_1(V - V_g) - f_2W + R_u, \quad (1)$$

$$\frac{dV}{dt} + \frac{\partial P}{\partial y} + \frac{\partial(G^{23}P)}{\partial \eta} = -f_1(U - U_g) + R_v, \quad (2)$$

$$\frac{dW}{dt} + \frac{1}{G^{1/2}} \frac{\partial P}{\partial \eta} + \frac{gP}{C_s^2} = f_2U + g \frac{G^{1/2} \bar{\rho} \theta'}{\bar{\theta}} + R_w \quad (3)$$

$$\frac{d\theta}{dt} = R_\theta, \quad \frac{ds}{dt} = R_s, \quad (4)$$

$$\frac{1}{C_s^2} \frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial}{\partial \eta} \left( G^{13}U + G^{23}V + \frac{1}{G^{1/2}}W \right) = \frac{\partial}{\partial t} \left( \frac{G^{1/2} \bar{\rho} \theta'}{\bar{\theta}} \right), \quad (5)$$

Here  $U = \bar{\rho} G^{1/2} u$ ,  $V = \bar{\rho} G^{1/2} v$ ,  $W = \bar{\rho} G^{1/2} w$ ,  $P = G^{1/2} p'$ , where  $p'$ ,  $\theta'$  are deviations from a basic state pressure  $\bar{p}$  and potential temperature  $\bar{\theta}$ ,  $s$  is the specific humidity,  $C_s$  is the sound wave speed,  $u_g$ ,  $v_g$  are components of a geostrophic wind representing the synoptic part of the pressure,  $\eta$  is a terrain-following coordinate transformation:

$$\eta = \frac{H(z - z_s)}{(H - z_s)}, \quad (6)$$

$z_s$  is the surface height,  $H$  is the height of the top of the model domain. Here  $H = \text{const}$ ,

$$G^{1/2} = 1 - z_s/H, \quad G^{13} = \frac{1}{G^{1/2}} \left( \frac{\eta}{H} - 1 \right) \frac{\partial z_s}{\partial x}, \quad G^{23} = \frac{1}{G^{1/2}} \left( \frac{\eta}{H} - 1 \right) \frac{\partial z_s}{\partial y}. \quad (7)$$

For an arbitrary function  $\varphi$

$$\frac{d\varphi}{dt} = \frac{\partial}{\partial t} + \frac{\partial u \varphi}{\partial x} + \frac{\partial v \varphi}{\partial y} + \frac{\partial \omega \varphi}{\partial \eta}, \quad (8)$$

where

$$\omega = \frac{1}{G^{1/2}} W + G^{13} u + G^{23} v. \quad (9)$$

The terms  $R_u$ ,  $R_v$ ,  $R_\omega$ ,  $R_\theta$ ,  $R_s$  refer to subgrid-scale processes. As a turbulence parameterization, we use the simple scheme:

$$K_m = \begin{cases} l^2 \sqrt{\frac{1}{2} D^2 (1 - \text{Ri})}, & \text{Ri} < 1, \\ 0, & \text{Ri} \geq 1, \\ \text{Ri} = \frac{g(d \ln \theta / dz)}{D^2/2}, & D = \nabla \underline{u} + \underline{u} \nabla. \end{cases}$$

A more detailed description of the model can be found, for instance, in[8]. We consider here a small-scale nonhydrostatic model developed for simulations mainly in meso- and microscales. In the present study, a two-dimensional finite-difference version of the model is employed[9].

### 3 Approximations

The advective terms in the model described above are approximated by difference operators based on conservation laws of the original equation system. The exact form of the equations is given in [8]. The time approximation is similar to that proposed in [9]. It is also described in paper [10]. The advective terms in the model described above are approximated by the following difference operators:

$$\delta_d \varphi = [\varphi(d + \Delta d/2) - \varphi(d - \Delta d/2)]/\Delta d, \quad (10)$$

$$\varphi^d = [\varphi(d + \Delta d/2) + \varphi(d - \Delta d/2)]/2, \quad (11)$$

$$ADV X = \delta_x(u^x(\rho^x u^x)) + \delta_y(v^x(\rho^x u)^y) + \delta_z(\varpi^x(\rho^x u)^z) \quad (12)$$

$$ADV Y = \delta_x(u^y(\rho^y v)^x) + \delta_y(v^y(\rho^y v)^y) + \delta_z(\varpi^y(\rho^y v)^z) \quad (13)$$

$$ADV Z = \delta_x(u^z(\rho^z w)^x) + \delta_y(v^z(\rho^z w)^y) + \delta_z(\varpi^z(\rho^z w)^z) \quad (14)$$

$$ADVT = \delta_x(u(\rho\Theta)^x) + \delta_y(v(\rho\Theta)^y) + \delta_z(\varpi(\rho\Theta)^z) \quad (15)$$

### 4 Numerical experiments on idealized front propagation over flat terrain

In this section the results of a series of calculations are presented to simulate the stratification and inversion effects on the surface pressure in the propagation of an atmospheric gravity current (cold front) over flat terrain. The model parameters are taken from paper [3]. The calculation domain is 25x25 km. In contrast to [3], where the front is generated by a volume of cold air, in the present study the front is initially given in the form of a step-function of 400 m in height. The front surface is treated by an efficient semi-Lagrangian scheme [11, 8]. Figure 1 shows the surface pressure in the middle of the simulation domain, at 12 km. Neutral stratification is adopted in the computation of the front. Figure 2 shows the results of calculations of the surface pressure at the same point 12 km under stable stratification with an inversion layer. Under stable stratification the front moves faster and shows an abrupt pressure jump at the point of observation [2]. The introduction of an inversion layer into the atmosphere increases the pressure further.

### 5 Conclusions

The change in stratification from neutral to stable in the propagation of a cold atmospheric front shows a time evolution of surface pressure that is in good agreement with the available observational data [3]. Under stable stratification the front moves faster and shows an abrupt pressure jump long before the front reaches the point of observation. This fact is in accordance with the present-day theory of atmospheric fronts [2]. The introduction of an inversion layer into the atmosphere increases the pressure further and its behavior is also in accordance with the simulation results obtained by a finite-difference model with initial generation of the front by an extended source of cold air [3].

### Список литературы

- [1] Pielke R. A. Mesoscale Meteorological Modeling. – Orlando: Academic Press, 1984.
- [2] Charba J. , Application of a gravity current model to analysis of squall-line gust fronts // Mon. Wea. Rev. – 1974. – Vol. 102, № 2. – P. 140–156.
- [3] Bischoff-Gauss, I. , Gross, G. , Numerical studies on cold fronts. Part 1: Gravity flows in a neutral and stratified atmosphere // Meteorol. Atmos. P. – 1989. – Vol. 40. – P. 150–158.

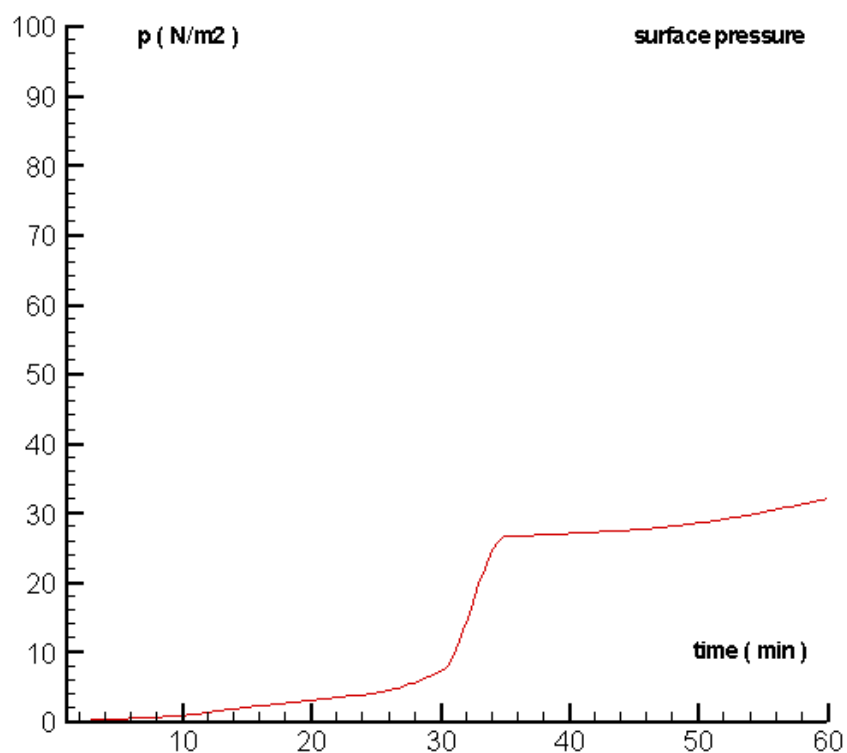


Рис. 1: Surface pressure at 12 km. Neutral stratification

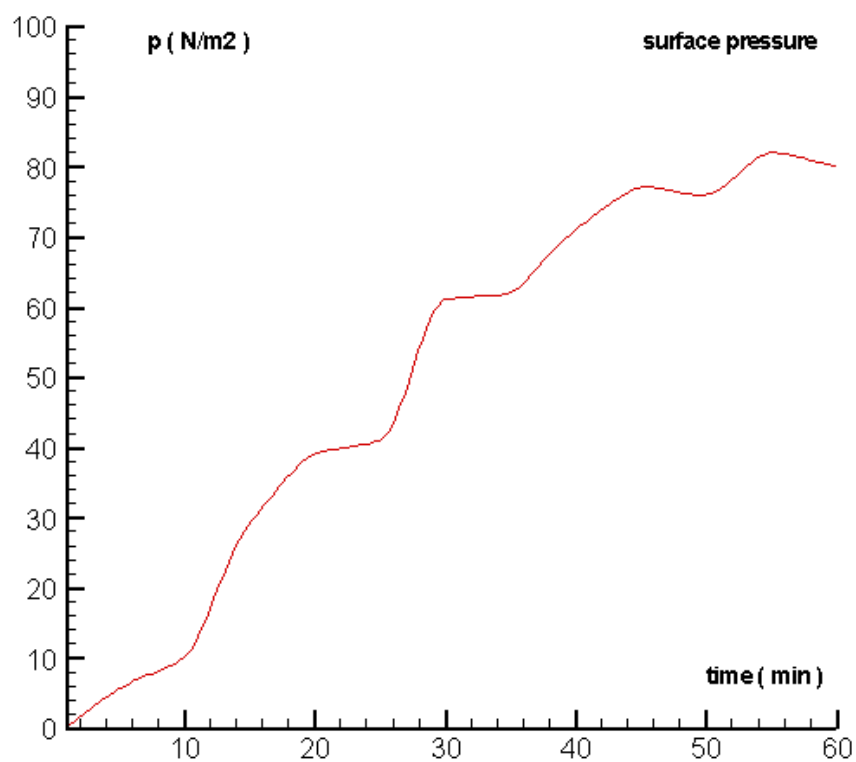


Рис. 2: Surface pressure at 12 km. Stable stratification with inversion

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- [4] Penenko V. V. , Aloyan A. E. Models and Methods for Environmental Problems. – Novosibirsk: Nauka, 1985 (In Russian).
- [5] Davies H. C. , On the orographic retardation of a cold front // Beitr. Phys. Atmos. – 1984. – Vol. 57. – P. 409–418.
- [6] Schumann U. Influence of mesoscale orography on idealized cold fronts // J. Atmos. Sci. – 1987. – Vol. 44, № 23. – P. 3423–3441.
- [7] Godunov S. K. Equations of Mathematical Physics. – Moscow: Nauka, 1978 (In Russian).
- [8] Yudin M. S. , Wilderotter K. Simulating atmospheric flows in the vicinity of a water basin // Computational Technologies. – 2006. – Vol. 11, № 3. – P. 128–134.
- [9] Ikawa M. Comparison of some schemes for nonhydrostatic models with orography // J. Meteor. Soc. Japan. – 1988. – Vol. 66, № 5. – P. 753–776.
- [10] Yudin M. S. , A numerical study of gravity waves in the atmosphere: smooth and steep orography effects // IOP Conference Series: Earth and Environmental Science. – 2016. – V. 48, № 1. – DOI [http://dx. doi. org/10. 1088/1755-1315/48/1/012024](http://dx.doi.org/10.1088/1755-1315/48/1/012024)
- [11] Ritchie H. Semi-Lagrangian advection on a Gaussian grid // Mon. Wea. Rev. – 1987. – Vol. 115. – P. 136–146.

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