Poroelastic effects influence range for radial hydraulic fracturing model

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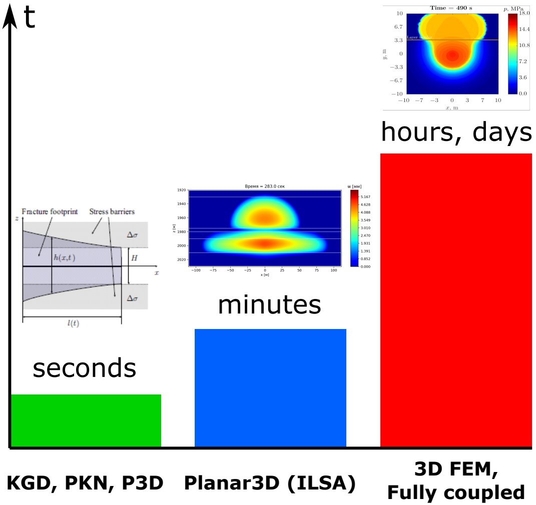
Fracturing Simulation Time

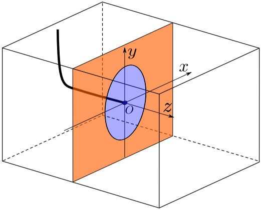


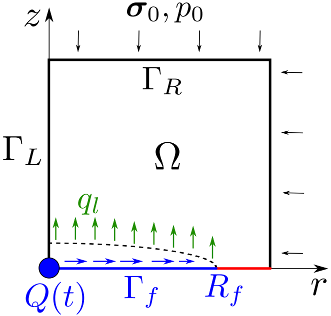
Problem Formulation

Biot's equations of poroelasticity

$$\operatorname{div} \boldsymbol{\sigma} = 0, \qquad \boldsymbol{\sigma} = \underbrace{\lambda \operatorname{div} \vec{u}\, \mathrm{I} + 2\mu\, \mathcal{E}(\vec{u})}_{\text{Elastic Stress}} - \underbrace{\alpha p\, \mathrm{I}}_{\text{Pore Pressure}}$$

$$S_\varepsilon \frac{\partial p}{\partial t} = \operatorname{div}\left(\frac{k_r}{\eta_r}\nabla p - \alpha \frac{\partial \vec{u}}{\partial t}\right)$$





$$\mathbf{\Gamma_R} : p = p_0, \quad \boldsymbol{\sigma}\langle\vec{n}\rangle = -\sigma_\infty \vec{n}$$

$$\mathbf{\Gamma_L} : \frac{\partial u_z}{\partial r} = 0, \quad u_r = 0, \quad \frac{\partial p}{\partial r} = 0$$

$$\mathbf{\Gamma_f} : \boldsymbol{\sigma}\langle\vec{n}\rangle = -(p - \sigma_{coh})\vec{n}, \quad u_z \geq 0$$

$$\mathbf{\Gamma_f} : \frac{\partial d}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{d^3}{12\eta_f}\frac{\partial p}{\partial r}\right) + \frac{k_r}{\eta_r}\frac{\partial p}{\partial z} + \frac{Q(t)}{2\pi r}\delta(r), \quad d = 2u_z.$$

Solution Method: Weak Formulation

$$\int_{\Omega} (\lambda \,\mathrm{div}\,(\vec{u}) - \alpha\, p)\,\mathrm{div}\,(\vec{\psi}) + 2\mu\,\mathcal{E}(\vec{u}) : \mathcal{E}(\vec{\psi})\, r\,drdz$$

$$- \int_{\Gamma_f} (p - \sigma_{coh} - \sigma_0)\psi_z\, r dr = 0, \quad \sigma_0 = \sigma_\infty - p_0,$$

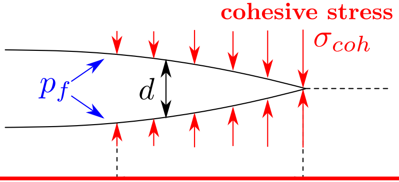
$$\int_{\Omega} S_\varepsilon \frac{\partial \mathbf{p}}{\partial \mathbf{t}} \varphi\, rdrdz + \int_{\Omega} \frac{k_r}{\eta_r} \nabla p \cdot \nabla \varphi\, rdrdz + \int_{\Omega} \alpha\, \frac{\partial}{\partial t}\Big(\mathrm{div}\,\vec{u}\Big)\varphi\, rdrdz +$$

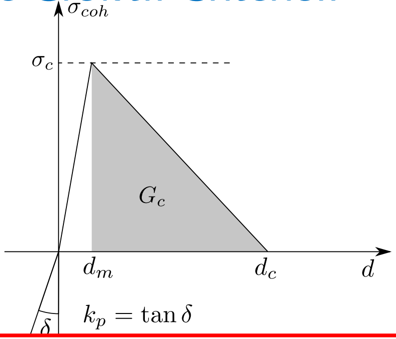
$$+ \int_{\Gamma_f} \frac{\partial \mathbf{u_z}}{\partial \mathbf{t}} \varphi\, rdr + \int_{\Gamma_f} \frac{\mathbf{u_z^3}}{3\eta_f} \frac{\partial p}{\partial r} \frac{\partial \varphi}{\partial r}\, rdrdz = \frac{\mathbf{Q(t)}}{4\pi} \varphi(0,0).$$

$$\begin{array}{lll} \Gamma_L : & \psi_r = 0, & u_r = 0 \\ \Gamma_R : & \varphi = 0, & p = 0 \end{array}$$

Solve via
Finite Element Method

Solution Method: Fracture Growth Criterion





$$+\int_{\Gamma_f} \sigma_{coh}(2u_z^{k+1})\,\psi_z\, r dr \approx$$

$$\int_{\Gamma_f} \Big(\sigma_{coh}(2u_z^k) + 2\frac{\partial \sigma_{coh}}{\partial(2u_z)}(2u_z^k)(u_z^{k+1} - u_z^k)\Big)\psi_z\, r dr$$

Reference Simulation Parameters

Parameter	Value
Young's Modulus, E	20 GPa
Poisson's Ratio, ν	0.24
Reservoir Permeability, k_r	$\mathbf{10^{-16}}$–$\mathbf{10^{-13}}$ m^2
Storativity, S_ε	$\mathbf{10^{-11}}$–$\mathbf{10^{-8}}$ Pa^{-1}
Biot Coefficient, α	$0, 0.75$
Closure Stress, σ_0	**1–100** MPa
Reservoir Fluid Viscosity, η_r	10^{-2} $\text{Pa} \cdot \text{s}$
Fracture Fluid Viscosity, η_f	$\mathbf{10^{-2}}$–$\mathbf{10^{-1}}$ $\text{Pa} \cdot \text{s}$
Pumping rate, Q	5×10^{-2} m^3/s

1D Filtration process

Adopting $\sigma_{zz} = (\lambda + 2\mu)\varepsilon_{zz} - \alpha p$

$$S_t \frac{\partial p}{\partial t} = \frac{\partial}{\partial z}\left(\frac{k_r}{\eta_r}\frac{\partial p}{\partial z}\right) - \cancel{\frac{\alpha}{\lambda + 2\mu}\frac{\partial \sigma_{zz}}{\partial t}}, \quad S_t = S_\varepsilon + \varkappa \frac{\alpha^2}{\lambda + 2\mu}, \tag{1}$$

Boundary and initial conditions:

$$p|_{t=0} = 0, \quad p|_{z=0} = P = const \tag{2}$$

Solution

$$p(t,z) = P \cdot erfc\left(\frac{z}{\ell_d(t)}\right), \quad c = \frac{k_r}{\eta_r \boldsymbol{S_t}},$$

Diffusion length $\boxed{\ell_d(t) = \sqrt{4ct}.}$

Carter formula

$$q_l = -\frac{k_r}{\eta_r}\frac{\partial p}{\partial z}\bigg|_{z=0} = \frac{C_L}{\sqrt{t}}, \quad C_L = P\sqrt{\frac{k_r}{\pi \eta_r}\boldsymbol{S_t}}, \tag{3}$$

higher **storativity** $\Rightarrow$ higher **leak-off**, but smaller **diffusion scale** ℓ_d.

1D Filtration Process

At point $z = \ell_d(t)$ pressure is dropped down to 15.72 % of the boundary level P. **Generalized diffusion scale:**

$$\tilde{\ell}_d(t) = \inf\{z : p(t,z) \leqslant \mathbf{0.1572}\, p(t,0)\} \qquad \ell_d(t) = \sqrt{4ct}$$

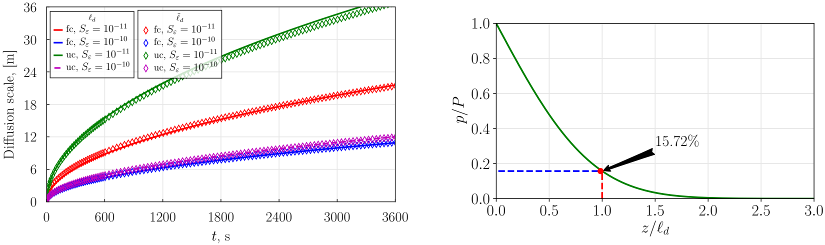
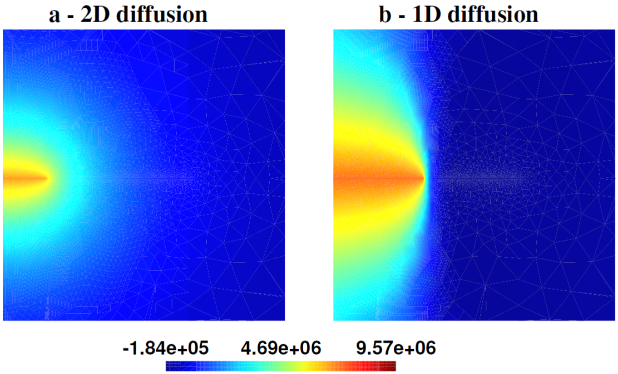


Figure: **Diffusion scales** for fully coupled (fc) $\alpha_e = \alpha_f = 0.75$ and uncoupled cases $\alpha_e = \alpha_f = 0.0$.

Other params: $k_r = 10^{-14}$ m^2, $\sigma_0 = 10$ MPa, $\eta_f = 0.01$ Pa·s

1d vs 3d diffision

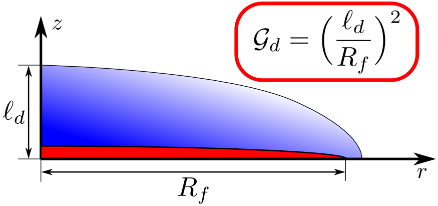


[Carrier, Granet, 2012]

- $\int\limits_{\Omega} \frac{k_r}{\eta_r}(\frac{\partial p}{\partial z}\frac{\partial \varphi}{\partial z} + \boldsymbol{\kappa}\frac{\partial p}{\partial r}\frac{\partial \varphi}{\partial r})rdrdz, \quad \kappa = 10^{-5}$ — anisotropic filtr. eq.
- Fracture front is faster than diffusion: $\ell_d \ll R_f$

1D to 3D diffusion transition

$\mathcal{G}_d^{crit} \approx 0.2 \Rightarrow$ fracture is 2.23 times faster than filtration front



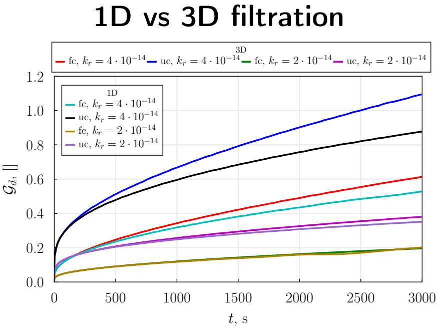


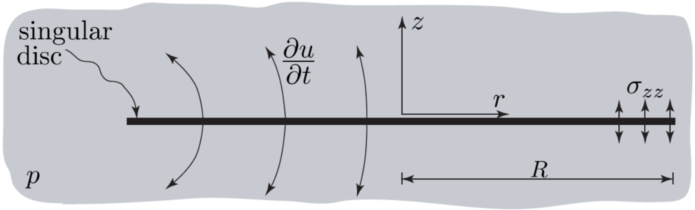
Figure: **Diffusion dimensionless complex** $\mathcal{G}_d$ in case of **3D diffusion** ($\kappa = 1$) and **1D diffusion** ($\kappa = 10^{-5}$) for fully coupled (fc) and uncoupled (uc) cases and different permeability ($k_r = 2 \times 10^{-14}$ m^2 and $k_r = 4 \times 10^{-14}$ m^2).
Other params: $S_\varepsilon = 10^{-11}$ Pa^{-1}, $\sigma_0 = 10$ MPa, $\eta_f = 0.1$ Pa·s

Backstress Green's functions

For constant pressure [Kovalyshen, 2010]

$$\sigma_b(t,r) = \int_0^t S_b(R_f(s), r, t-s)\boldsymbol{p}(s)ds,$$

$S_b(R_f, r, t)$ — **vertical stress** σ_{zz} generated by **instantaneous unit pressure impulse** along disk $\{r \leqslant R_f, z = 0\}$.



Backstress dimensionless complex

Scaling:

$$\sigma_b(t,r) = (p^*(t)-\sigma_0)\Sigma(t,r), \quad S_b(R_f,r,t) = \frac{\eta}{T_{R_f}}\,\Xi\!\left(\frac{r}{R_f},\frac{t}{T_{R_f}}\right),$$

$$T_{R_f} = \frac{R_f^2}{4c}, \quad \eta = \alpha\frac{1-2\nu}{2-2\nu}, \quad \mathcal{G}_d(t) = \frac{\ell_d^2(t)}{R_f^2(t)},$$

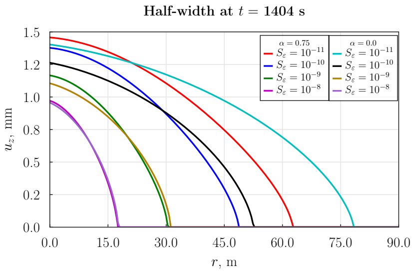
$$\Sigma(t,s) = \mathcal{G}_{bs}(t)\int\limits_0^1 \frac{R_f^2(t)}{R_f^2(ts)}\Xi\!\left(\frac{r}{R_f(ts)},\mathcal{G}_d(1-s)\frac{R_f^2(t)}{R_f^2(ts)}\right)\frac{p^*(ts)}{p^*(t)}ds,$$

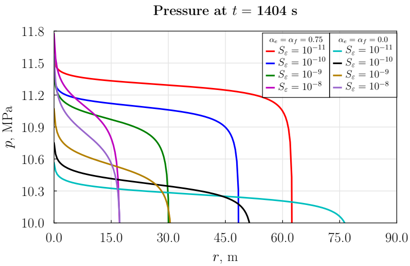
$$\boxed{\mathcal{G}_{bs}(t) = \eta\mathcal{G}_d(t)\left(1+\frac{\sigma_0}{p^*(t)-\sigma_0}\right)}$$

$p^*(t)$ — **mean pressure value**

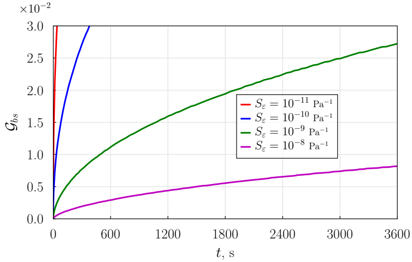
Backstress dimensionless complex

Poroelastic effecs



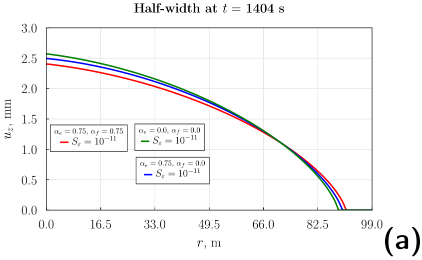


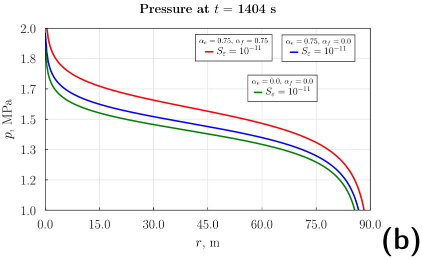
$$\mathcal{G}_{bs}^{crit} = 10^{-3}$$



Param.	Value
k_r	10^{-14} m^2
σ_0	10 MPa
η_f	10^{-2} Pa · s

Deformation domination



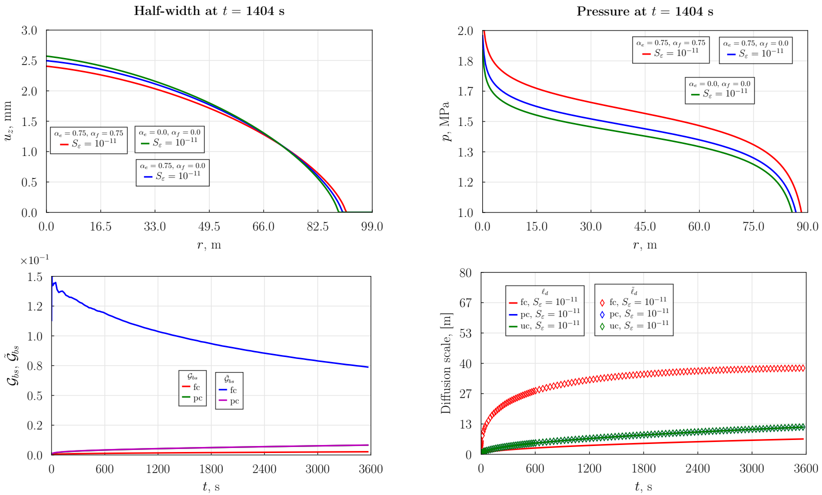


Param.	Value
k_r	10^{-15} m^2
σ_0	1 MPa
η_f	10^{-1} Pa · s

$$C_L = \sigma_0 \sqrt{\frac{k_r}{\pi \eta_r} S_t} \approx 10^{-6} \text{m}/\sqrt{\text{s}}$$

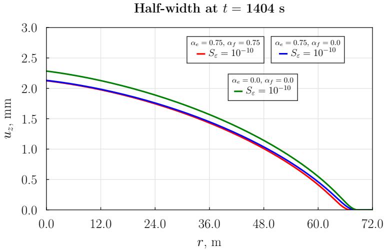
almost **Storage-viscosity regime**

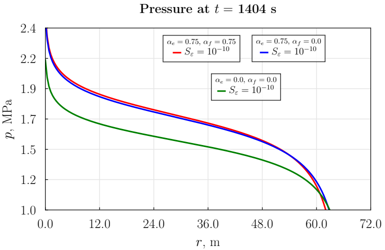
Deformation domination

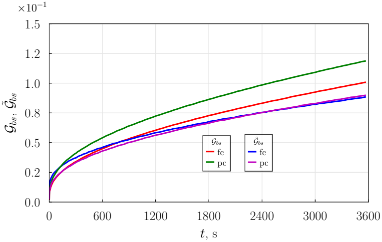


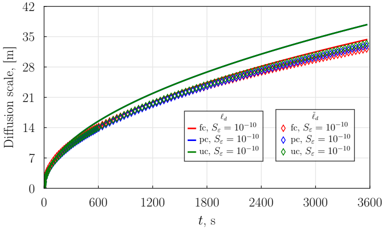
$\tilde{\mathcal{G}}_d(t)$ **reflects** pressure diffusion **better**

Deformation domination (weak effect)









Other parameters: $k_r = 10^{-13}$ m^2, $S_\varepsilon = 10^{-10}$ Pa^{-1}.

Conclusion

- Algorithm of radial **hydraulic fracture growth** in 3D poroelastic medium is constructed
  - Simple to implement
  - Robust in wide parameters range
- Dimensionless complexes reflecting **diffusion regime** and **backstress** are considered. Critical values are estimated
- **Deformation** domination diffusion case is **investigated**