Mathematical modeling of the consumer loan market in Russia

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In this talk we present the mathematical description of the economic behavior of a rational household in consumer loan market. The modeling of the economic behavior of households is based on the concept of a rational representative economic agent and arises to F. Ramsey. The model is formalized as an optimal control problem on a finite time horizon. The household maximizes discounted consumption with constant risk aversion, managing the dynamics of its expenditures depending on the current parameters of the economic situation and the behavioral characteristics of the household itself. We consider an imperfect market when the interest rate on loans differs from the interest rate on deposits. The difference in interest rates on loans and deposits leads to non-smoothness of the right-hand side of the differential equation for the phase variable. This motivates to use the Pontryagin maximum principle in the form of F. Clark. Applying it, we obtain an area where the household does not interact with the banking system, the special regimes arise. If we tend the time horizon of an optimal control problem to infinity, it is possible to construct a synthesis. The synthesis allows us to determine an optimal control depending on the current value of the phase variable and the parameters of the economic situation. It depends on current interest rates and on the behavioral characteristics of a representative household. We develop and investigate a new model for the formation of interest rates on consumer loans based on an analysis of commercial interests and the logic of behavior of commercial banks. The model assumes that the borrowersвЂ™ incomes are described by a geometric Brownian motion. The commercial banks assess the default risk of borrowers. According to the Feynman–Kac formula, the assessment is reduced to solving a boundary value problem for partial differential equations. An analytical solution to this problem is constructed. It is possible to reduce the solution of the boundary value problem to the Cauchy problem for the heat equation with an external source and obtain a risk assessment in analytical form with a help of the Abel equation. The models of economic behavior of households in the consumer loan market and behavior of commercial banks are identified based on Russian statistics. A specialized software has been developed to analyze the demand for consumer credit. With its help, the problems of the consumer lending market in Russia are analyzed.

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