About One Approach to the Planning in Retail with the Stochastic Parameters

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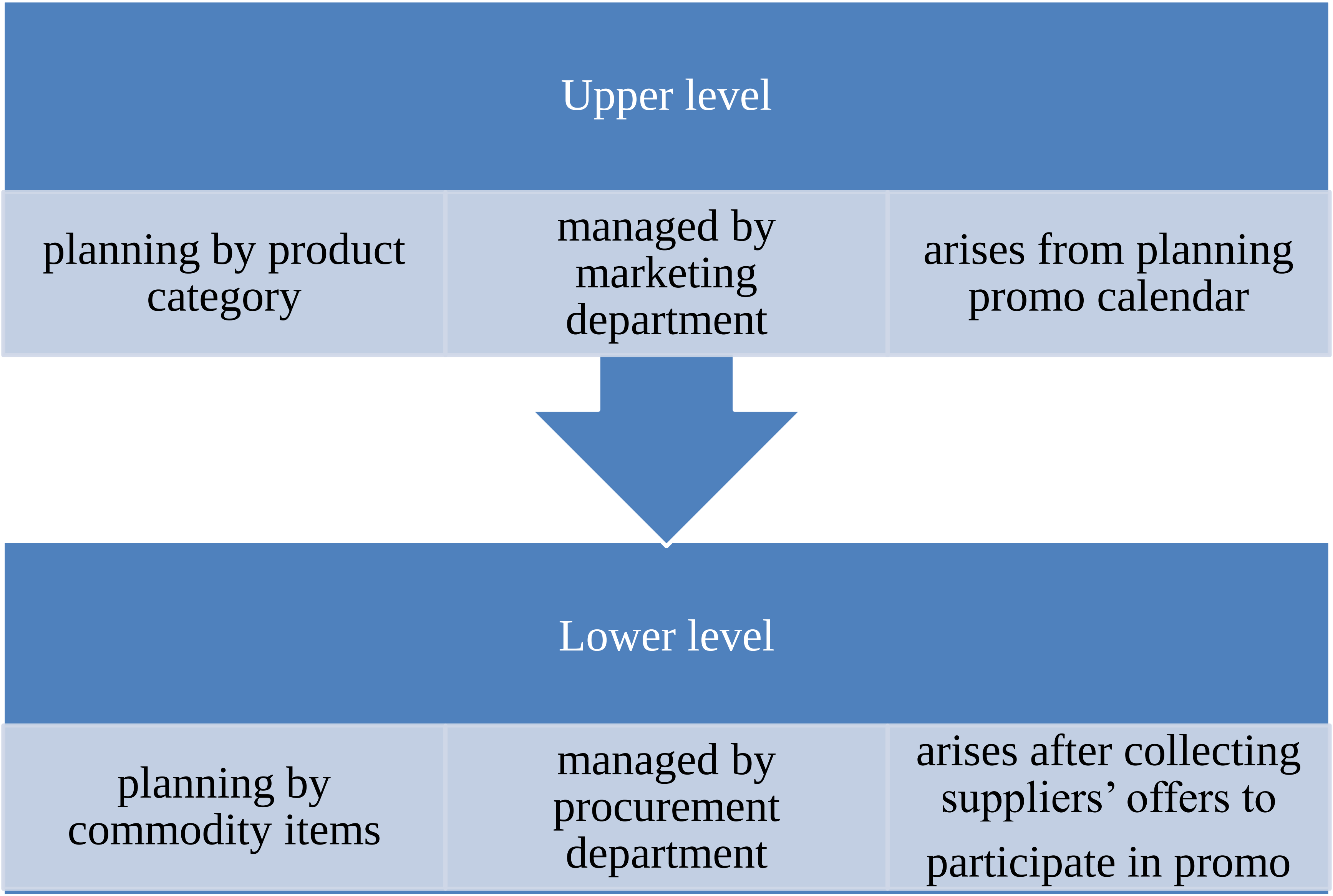
INTRODUCTION

- Retail requires special attention to the effective promo activities
- We propose a new model of promo planning that allows us to create an optimal annual plan for promotional activities in discount food store chain
- **Promo** is a special price offer for the buyers.
- **In-out promotion** is a retail promotion when the products are not selected from the regular assortment, but are introduced to the assortment only for the promotional period.
- It is also assumed that the duration of the promo is at least **one week**.


Upper level
planning by product category
managed by marketing department
arises from planning promo calendar
Lower level
planning by commodity items
managed by procurement department
arises after collecting suppliers’ offers to participate in promo

Notation

- a_{ij} – net present gross margin of *i* category on *j* week;
- ν_i– average period between the purchases of *i* category product;
- y_{ij} – binary variables are equal to 1, if promo is required for *i* category on *j* week, otherwise are equal to 0;
- v_k– physical volume of *k* product (in m^3);
- q_{kj} – size of *k* product delivery for *j* week (pcs.);
- b_{kj} – gross margin of k product on j week;
- x_{kj}– binary variables, possessing the value of 1, if promo is required for k product on j week, otherwise are equal to 0;

Notation

- $Nlot$ – total number of suppliers' offers;
- $Ncat$– total number of product categories;
- S– max number of product categories involved in the promo at the same time;
- V_j – maximum physical volume for promo in the retail chain on *j* week (in m^3);
- L_j– minimum number of products in promo for *j* week;
- μ_{kj}– indicator of product nesting into the category, is equal to 1, if *k* product belongs to *i* category, otherwise is equal to 0.

BASIC DESCRIPTION OF THE MODEL

$$\mathbf{M}\,(\sum_{j=1}^{52}(\sum_{i=1}^{Ncat} a_{ij} * y_{ij})) \Rightarrow \max_{y_{ij}} \qquad (1)$$

Subject to

$$\mathbf{P}\,(\sum_{j=1}^{52} y_{ij} \le \frac{52}{\nu_i} * \omega_j) \ge (1-\alpha_i), i = 1,\ldots,Ncat \qquad (2)$$

$$\sum_{i=1}^{Ncat} y_{ij} \le S,\ j = 1,\ldots,Ncat \qquad (3)$$

$$\sum_{j=i}^{l+\nu_{ij}-1} y_{ij} \le 1, l = 1,\ldots,52 \qquad (4)$$

$$\sum_{k=1}^{Nlot} x_{kj}^{*} * \mu_{ki} * y_{ij} \ge 1, i = 1,\ldots,Ncat,\ j = 1,\ldots,52 \qquad (5)$$

$$y_{ij} \in \{0,1\}, i = 1,\ldots,Ncat,\ j = 1,\ldots,52 \qquad (6)$$

BASIC DESCRIPTION OF THE MODEL

$$\mathbf{M}\left(\sum_{k=1}^{Nlot}\left(\sum_{j=1}^{52} b_{kj} * x_{kj}\right)\right) \Rightarrow \max_{x_{kj}} \qquad (7)$$

Subject to

$$\mathbf{P}\left(\sum_{k=1}^{Nlot} x_{kj} * q_{kj} * v_k \leq \xi_j * V_j\right) \geq (1-\beta),\ j = 1, \dots, 52 \qquad (8)$$

$$\sum_{k=1}^{Nlot} x_{kj} \leq L_j,\ j = 1, \dots, 52 \qquad (9)$$

$$\sum_{k=1}^{Nlot} x_{kj} * \mu_{ki} \leq Nlot * y_{ij},\ i = 1, \dots, Ncat,\ j = 1, \dots, 52 \qquad (10)$$

$$x_{kj} \in \{0,1\},\ k = 1, \dots, Nlot,\ j = 1, \dots, 52 \qquad (11)$$

BASIC DESCRIPTION OF THE MODEL

- α, β – precision limits;
- ω, ξ– random variables with distributions ϕ, ψ;
- (1) can be supplemented by the constants

$$\sum_{k=1}^{Nlot} x^{*}_{kj} * b_{kj} * \mu_{kj},\ j = 1,\ldots,Ncat$$

in order to evaluate the leader's result more adequately . This term characterizes the sales gain of the category from in-out promotions on *j* week, taking into account the proposed products at the lower level. In numerical experiments this value was taken into account.

REDUCTION OF THE STOCHASTIC PROBLEM

- Let us consider the distribution densities of random variables:

$$\phi_i(\omega_i) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \phi(\omega_1, \dots, \omega_m) \prod_{i \neq j} d\omega_j \qquad (12)$$

$$\psi_i(\xi_i) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \psi(\xi_1, \dots, \xi_m) \prod_{i \neq j} d\xi_j \qquad (13)$$

- We shall find the roots $\widetilde{\omega_i}$, $\widetilde{\xi_i}$ from the integral equations of types

$$\int_{\widetilde{\omega_i}}^{\infty} \phi_i(\omega_i) d\omega_i \qquad (14)$$

$$\int_{\widetilde{\omega_i}}^{\infty} \psi_i(\xi_i) d\xi_i \qquad (15)$$

- If more than one root is found, we choose the maximum from feasible solutions

REDUCTION OF THE STOCHASTIC PROBLEM

After that, the problem (1) – (11) can be represented as a bilevel deterministic problem of linear mathematical programming, where the objectives (1),(7) and the constraints (2),(8) become (16), (17) and (18), (19) respectively:

- $\sum_{j=1}^{52}(\sum_{i=1}^{Ncat} a_{ij} * y_{ij}) \to \max_{y_{ij}}$ (16)
- $\sum_{j=1}^{52}(\sum_{k=1}^{Nlot} b_{kj} * x_{kj}) \to \max_{x_{kj}}$ (17)
- $\sum_{j=1}^{52} y_{ij} \leq \frac{52}{v_i} * \widetilde{\omega_j}$, $i=1, \ldots, Ncat$ (18)
- $\sum_{k=1}^{Nlot} x_{kj} * q_{kj} * v_{kj} \leq \widetilde{\xi_i} * V_j$, $j=1, \ldots, 52$ (19)

Example: ϕ, ψ have density function of Simpson's distribution [0;2]

$\alpha \geq 0,5$:

- $\int_{\widetilde{\omega_i}}^{1} x\,dx + \int_{1}^{2} (-x+2)\,dx = \alpha$
- $-\frac{\widetilde{\omega_i}^2}{2} + 1 = \alpha$
- $\widetilde{\omega_i} = \sqrt{-2(\alpha - 1)}$

$\alpha < 0,5$:

- $\int_{\widetilde{\omega_i}}^{2} (-x+2)\,dx = \alpha$
- $\widetilde{\omega_i} = 2 \pm \sqrt{2\alpha}$

α	$\widetilde{\omega_i}$
0.95	0,316
0.9	0,447
0,8	0,632
0,7	0,775
0,5	1
0,4	1,106

Construction of Deterministic Optimization Problems Sequence

1. Generation of *N* pseudorandom values ω^0, ξ^0
2. for $d = 1, \dots, N$ do
   a) Replace the random variables ω, ξ in restrictions (2),(8) by ω^0, ξ^0
   b) Since we fix the values of the random variables (1),(7) and (2),(8) become (16),(17) and (18),(19) respectively

$$\sum_{j=1}^{52} y_{ij} \le \frac{52}{v_i} * \omega_j{}^0, i = 1, \dots, Ncat \quad (18)$$

$$\sum_{k=1}^{Nlot} x_{kj} * q_{kj} * v_k \le \xi_j{}^0 * V_j, j = 1, \dots, 52 \quad (19)$$

   c) Solve deterministic problem

3. find d such that:

$F = \left| \frac{\sum_{l=1}^{N} F_l}{N} - F_d \right| \Rightarrow min$ and $U = \left| \frac{\sum_{l=1}^{N} U_l}{N} - U_d \right| \Rightarrow min$ where F_l, U_l values of objectives (16), (17)

4. Vectors *y*, *x* corresponding to *F* and *U* are problem solutions

Deterministic Problem Solving

Obtained deterministic problems are proved to be *NP-hard*. In order to solve these problems, we can use various methods including the heuristic one

Direct enumeration:

All possible variants of vector components values y are searched through. The number of variants of enumeration does not exceed $2^{52*Ncat}$.

We shall note that constraints (18) significantly reduce the number of possible variants *y*.

Deterministic Problem Solving

«Game theoretic» approach:

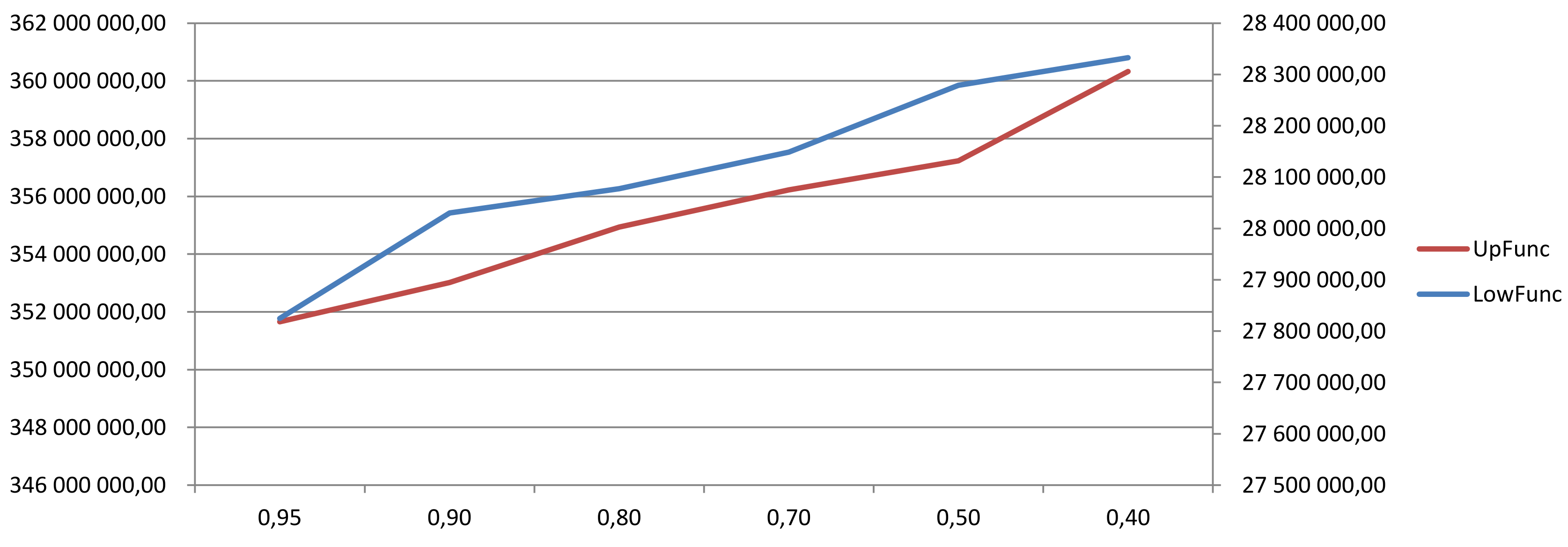
1. Problem of lower level supplemented by the constraints (18) and $\sum_{j=1}^{52}\left(\sum_{i=1}^{Ncat} a_{ij} * y_{ij}\right) \geq M$
2. Iteration process of maximum M search is carried out, according to which the Boolean programming problem with linear constraints contains an optimal solution of "Leader-Follower" game.

Numerical experiment

Experiments were conducted on the basis of the actual data of LLC “NSK Holdi”. This example contained the information about the 47 product categories and 3119 offers from suppliers.

We use "Reduction to the deterministic problem" method and took Simpson distribution on [0;2].

Note that the objective of upper level was supplemented by constants $\sum_{k=1}^{Nlot} x_{kj}^{*} * b_{kj} * \mu_{kj}, j = 1, \ldots, Ncat.$



Conclusion

A new planning model for food retail is constructed.

We provided the approaches to the solution of linear stochastic programming problem, as well as of arising bilevel deterministic linear problems.

We conducted numerical experiments on real data.

Thank you for your attention

Questions?