LAVRENTYEV INSTITUTE OF HYDRODYNAMICS OF SB RAS
MATHEMATICAL CENTER IN AKADEMGORODOK
NOVOSIBIRSK STATE UNIVERSITY

Russia-Japan-India Workshop

MATHEMATICAL ANALYSIS OF FRACTURE PHENOMENA AND ITS APPLICATIONS

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ABSTRACTS

Novosibirsk
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The mathematical foundation of fracture mechanics has seen considerable advances in the last years. This field of science covers a big variety of exciting topics, including propagation of cracks, equilibrium of structures with thin inclusions in the presence of delaminations, frictional contact problems, inverse and control problems. The aim of the international workshop 'Mathematical analysis of fracture phenomena for elastic structures and its applications' is to bring together researchers working on different aspects of these issues. The workshop provides a platform for researchers to communicate, discuss, and exchange ideas under the common theme of fracture phenomena.

The Workshop topics:

- elasticity problems
- modeling of composite materials
- fracture mechanics
- study of mathematical models for solids with defects
- asymptotic and multiscale analysis
- optimal shape design
- inverse problems

The first Workshop 'Mathematical analysis of fracture phenomena for elastic structures and its applications' was hosted in Novosibirsk in November 2019 due to the collaboration of researchers from Japan and Russia. The Second Russia-Japan Workshop was held in December 2020. The geography of the workshop participants was expanded: researchers from Russia, Japan, Germany, Austria and Czech Republic were involved. Another meeting happen in December 2021 when The Third Russia-Japan Workshop took place.

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MATHEMATICAL MODELLING ON LINEARIZED VISCOELASTICITY WITH FRACTIONAL DERIVATIVES

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In this talk, we discuss mathematical modelling on linearized viscoelasticity under infinitesimal strain. Firstly, we introduce classical models and their creep compliance and relaxation modulus. Next, one of generalization of the models is a viscoelastic fractional model by use of fractional calculus. We explain the constitutive relation for such models, which have possibility to describe a wide variety of viscoelastic materials. Lastly, we consider to extend classical models to nonlinear viscoelastic models and introduce our recent results.

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WELL-POSEDNESS OF THE GOVERNING EQUATIONS FOR NONLINEAR ELASTIC MODEL IN WHICH BOTH STRESS AND STRAIN APPEAR LINEARLY

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The response of a body described by a nonlinear elastic constitutive relation, in which both stress and strain appear linearly, is studied. The constitutive relation stems from a class of implicit relations between the histories of the stress and the relative deformation gradient. A-priori thresholding is enforced through the material moduli that ensures that the displacement gradient remains small. Moreover, the thresholding procedure ensures that the solution does not blow-up in finite time. The resulting mixed variational problem consists of a quasi-static equilibrium equation and the nonlinear material response supported by mixed Dirichlet–Neumann boundary conditions. The problem is studied for well-posedness within the theory of coercive and maximal monotone graphs. The theoretical results are supported by computer simulation of representative examples solved numerically with respect to monotone loading both with and without thresholding.

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ASYMPTOTICS OF WEAKLY CURVED ANISOTROPIC INCLUSIONS IN ELASTIC BODY

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We study boundary value problems describing an equilibrium of two-dimensional elastic bodies with anisotropic weakly curved inclusions in a presence of delaminations. The presence of delaminations means the existence of cracks between the inclusions and the surrounding elastic body. Nonlinear boundary conditions of the inequality type are specified on the crack faces, providing the mutual nonpenetration of the faces. This leads to boundary value problems with unknown contact domains. Limit transitions are analyzed for the rigidity parameters of thin inclusions tending to infinity. In particular, limit models are investigated.

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SIGNORINI TYPE CONTACT PROBLEMS FOR AN NONHOMOGENEOUS PLATE SUBJECT TO A POINTWISE NONPENETRATION CONDITION

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Two nonlinear mathematical models describing the stress-strain states of Timoshenko plates containing thin rigid inclusions are proposed. The novelty of corresponding problems is determined by new pointwise conditions describing the nonpenetration of inclusion points. In addition, we prove the possibility of using the fictitious domain method for the new proposed problems under consideration. In doing so, we construct a family of auxiliary crack problems formulated in an enlarged domain. Each problem of the family depends on positive parameter $\lambda \in (0, \lambda_0]$ which affects elastic modulus. We prove that the solutions of these problems converge to the solution of the contact problem weakly in the Sobolev space H^1. The previous works related to feasibility of the method of fictitious domain method in the framework of Signorini type problems have been obtained for cases where nonpenetration conditions were imposed on curves or surfaces with positive measures [1, 2, 3, 4], etc.

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ON MODELING THE JUNCTION OF THIN ANISOTROPIC INCLUSIONS IN A TWO-DIMENSIONAL ELASTIC BODY

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The research concerns a junction problems for two thin anisotropic inclusions inside a two-dimensional elastic body. The inclusions model is characterized by a given structure of displacements functions and rotation angles. Several cases of junction geometry and junction nature are considered. From the geometric point of view, junction can take place at the end points of inclusions or at the internal point of one of them. From the point of view of the junction nature, cases of contact between two unconnected inclusions are considered, as well as junction of inclusions under conditions of perfect adhesion. It is assumed that one of the inclusions delaminates from the elastic matrix forming a crack. Due to the presence of a crack, the elastic body occupies a domain with a cut, while on the cut edges, as on a part of the boundary, boundary conditions of the form of inequalities are set. These conditions exclude mutual penetration of the crack edges into each other. At the same time, such a formulation leads to the nonlinearity of the problem and the need to use additional mathematical methods to construct an algorithm for the numerical solution of the problem [1-4]. The problem is posed as a variational one, and a complete differential formulation in the form of a boundary value problem is also obtained, including the junction conditions at a joint point of inclusions. The equivalence of the variational and differential formulations of the problem is proved under the condition of sufficient smoothness of the solutions. For the numerical solving of the problem in a domain with a cut, a variational formulation is used using the domain decomposition method and the Uzawa algorithm. Examples of a computational experiment are given.

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OBSTACLE DETECTION IN STOKES FLUID FLOW USING A NOVEL SHAPE OPTIMIZATION APPROACH

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We propose a novel shape optimization approach to identify an obstacle immersed in a fluid within a larger bounded domain, using boundary measurements on the accessible surface; see [1]. The fluid motion is governed by the Stokes equations. Specifically, we address the inverse problem of obstacle identification by employing shape optimization techniques through the coupled complex boundary method. This method transforms the over-specified problem into a complex boundary value problem by introducing a complex Robin boundary condition that couples the Dirichlet and Neumann boundary conditions along the accessible boundary. To reconstruct the unknown boundary, we minimize a cost function based on the imaginary part of the solution across the entire domain. The shape gradient of this cost function is then computed and used to iteratively solve the optimization problem via a Sobolev gradient descent algorithm. We demonstrate the feasibility of this method through numerical experiments in both two and three spatial dimensions. This work is a collaboration with Hirofumi Notsu (Kanazawa University, Japan) and Lekbir Afraites (Université Sultan Moulay Slimane, Morocco).

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THE HOMOGENIZED DYNAMICAL MODEL OF A THERMOELASTIC COMPOSITE REINFORCED BY THIN FILAMENTS

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The dynamical problem of linear thermoelasticity for a body with incorporated thin rectilinear inclusions is studied. It is assumed that the inclusions (i.e. filaments and threads) are parallel to each other and the problem contains a small parameter $\varepsilon > 0$, which characterizes the distance between two neighbouring inclusions. Using the two-scale convergence approach, we find the limiting problem as ε goes to 0. As a result, we get a well-posed homogenized model of an anisotropic inhomogeneous body with effective characteristics inheriting thermomechanical properties of inclusions.

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RECONSTRUCTION OF THE INCLUSION FROM BOUNDARY MEASUREMENTS BY USING NEURAL NETWORKS AND ENCLOSURE METHOD

Takanori Ide, Samuli Siltanen

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Electrical Impedance Tomography (EIT) is non-destructive imaging method that recovers image of the inclusions from boundary measurements. We propose novel method to reconstruct the inclusion from boundary measurements by using neural networks and enclosure method. As the application of proposed method, we will demonstrate numerical computation.

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SCATTERING PROBLEM ON A QUANTUM GRAPH

Igor Trushin

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In this talk we discuss the problem of the characterization of the scattering data for a Sturm-Liouvile operator on the quantum graphs. Partial results in the case of of infinite graph containing circle presented. Results of this presentation were obtained jointly with Prof. K.Mochizuki.

INVERSE OBSTACLE SCATTERING PROBLEMS

Guang-Hui Hu[1] and Manmohan Vashisth[2]

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The talk is concerned with the inverse problems related to acoustic scattering. We shall discuss our obtained results related to the unique recovery of the shape of an obstacle from the knowledge of a far-field pattern coming from a single incident plane wave. The talk is based on joint work with Prof. Guang-Hui Hu, Nankai University, China.

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