Energy Localization and Solitons in Nonlinear Discrete Systems

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This talk will overview the recent progress in the study of the energy localization and solitons in a variety of nonlinear physical systems, in particular those where the effects of discreteness and periodicity become important. This general presentation will cover: (i) optical spatial solitons in waveguide arrays and photonic lattices, including the most recent observation of polychromatic solitons generated by a supercontinuum source, (ii) control of matter waves in Bose-Einstein condensates loaded onto periodically varying one- and two-dimensional optical lattices, and (iii) energy localization in graphene nanoribbons and carbon nanotubes.

First, I will emphasize the most important recent advances in nonlinear optics where many of novel theoretical findings have been verified in experiment. This includes the observation of surface solitons in one- and two-dimensional photonic lattices, the observation of polychromatic "rainbow" gap solitons in photonic lattices generated by a supercontinuum source, the generation of topologically stable spatially localized multivortex solitons, etc. One of the recent concepts in the theory of nonlinear waves is associated with a novel type of broad nonlinear states which appear in the gaps of the bandgap spectra of periodic systems. These localized states cannot be treated by familiar multi-scale asymptotic expansion techniques, and they can be better understood as truncated nonlinear Bloch waves. I will demonstrate that these self-trapped localized nonlinear modes can be found in one-, two-, and three-dimensional periodic potentials, and they have been readily observed in experiments on nonlinear self-trapping of matter waves in one-dimensional optical lattices.

Next, I will review our recent results on the dynamics of bright solitons in BEC with attractive atomic interactions perturbed by a weak bi-chromatic optical lattice potential. The lattice depth is a bi-periodic function of time with a zero mean, which realizes a flashing ratchet for matter-wave solitons. We find that the average velocity of a soliton and the soliton current induced by the ratchet depend on the number of atoms in the soliton. As a consequence, soliton transport can be induced through scattering of different solitons. In the regime when matter-wave solitons are narrow compared to the lattice period the dynamics is well described by the Hamiltonian theory.

Finally, I will discuss the energy localization in carbon nanotubes and demonstrate the existence of spatially localized nonlinear modes in the form of discrete breathers. In nanotubes with the chirality index ***(m,0)*** there exist three types of discrete breathers associated with longitudinal, radial, and torsion anharmonic vibrations, however only twisting breathers survive in a curved geometry remaining long-lived modes even in the presence of thermal fluctuations