New Equations for Simulation of the Nonlinear Waves Interaction on a Free Surface of the Shallow Water

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Formally the equations of the Boussinesq-type allow studying a collision of two plane waves with small but finite amplitudes running towards each other. However actually such equations may be derived only for weakly nonlinear disturbances propagating mainly in an arbitrary but one horizontal direction.

This paper deals with an interaction of moderately long plane localized waves running simultaneously both in the direction the horizontal coordinate x growth and in the opposite side above a gently sloping bottom. The auxiliary function ψ is introduced so that the mass conservation law is applied identically. For this function the following evolution equation is obtained from the basic equation of the model proposed in [1]:

$$\frac{\partial^2\psi}{\partial t^2} - gh\frac{\partial^2\psi}{\partial x^2} - \frac{\partial}{\partial x}\left[\frac{g}{2}\left(\frac{\partial\psi}{\partial x}\right)^2 + \frac{1}{h}\left(\frac{\partial\psi}{\partial t}\right)^2\right] - \beta\frac{\partial^4\psi}{\partial t^2\partial x^2} = 0 .$$

Here t is the time, g is the free fall acceleration, h is the equlibrium water depth, $\beta = h^2(1/3-1/\mathrm{Bo})$, the Bond number $\mathrm{Bo} = \rho g h^2/\sigma$, ρ is the liquid density, and σ is the surface tension.

Using this equation we analytically found an approximate solution for a head-on collision of two localized waves. It was shown perturbatively that, in contrast to the modified Boussinesq equation, the newly derived equation can describe the inelastic interaction of counter propagating waves [2]. The analytical solutions of the problem of the collision of two solitons in a layer of a fluid above a horizontal bottom (see Fig. 1) and the numerical solutions of a similar problem for a gently sloping bottom was found. The fluid is almost completely stopped at the time of the maximum interaction of solitons with initially equal amplitudes. Consequently, the kinetic energy of the layer vanishes and the potential energy is correspondingly maximal.

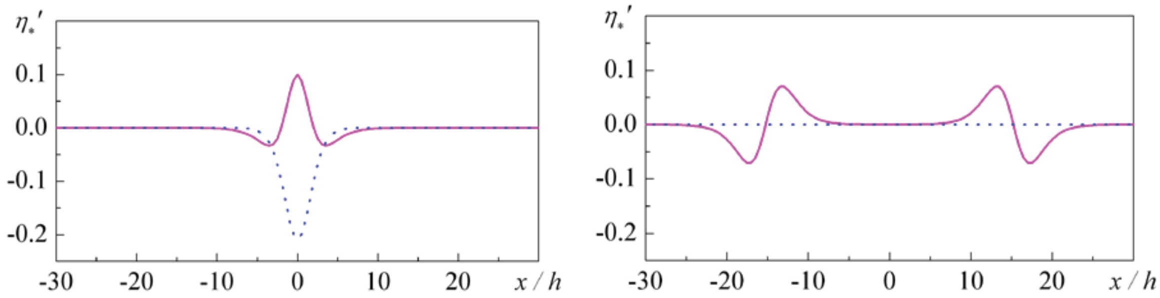

Fig. 1.

Let us recall that the derivation of the modified Boussinesq equation assumes that the velocity of the fluid is proportional to the perturbation of the free surface; i.e., the kinetic energy of the layer reaches the maximum at the time of the maximum interaction of the waves. This explains why the calculation with the use of the modified Boussinesq equation underestimates the amplitude by approximately 15%.

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References

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