Turbulent Scaling in the Viscous Sublayer

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Based on matched asymptotic expansion method the new scaling relations for the mean velocity and Reynolds shear stress in the viscous sublayer are proposed, which generalize the classical scaling related with the logarithmic law of the wall. It is found that the matching condition for the mean velocity in the overlap region is equivalent to Vincze's functional equation and has the solution in the form of power law.

Scaling of wall turbulence at high Reynolds numbers is in the focus of research almost from the start of wall turbulence studies inspired by classical works of L.Prandtl, G.I.Taylor, Th. von Kármán, A.A.Izakson and C.B.Millikan [1]. For the canonical wall-bounded turbulent flows the classical scaling is based on the two main assumptions. First is the two-layer structure of wall turbulence consisting of the outer layer and inner, or viscous, sublayer. Second is the friction velocity as the scale for the velocity fluctuations and mean velocity difference in both viscous sublayer and outer layer. Then, following Izakson-Millikan arguments, the logarithmic law for the mean velocity and scaling relations for the statistical moments of velocity fluctuations can be derived [1,2].

In the present work the method of matched asymptotic expansions is applied to the problem of the scaling of wall turbulence. In contrast to previous works the present analysis is not restricted to the propositions about the friction velocity as the proper velocity scale for inner and outer layer. It is shown that the mean momentum balance equation in the viscous sublayer admits arbitrary velocity scale for the mean velocity, not only the friction velocity. The new inner scaling is proposed, in which the new relevant parameter – the third-order wall-normal derivative of the Reynolds shear stress at the wall Λ is added. Proposed scaling gives the better collapse for both the mean velocity and Reynolds shear stresses profiles in the viscous sublayer at different Reynolds numbers. It is found that the matching condition for the mean velocity in the overlap region is equivalent to Vincze's functional equation and has the solution in the form of power law. Power law exponent tends to zero in the limit of infinite Reynolds number and only in the case of complete similarity of Λ and outer velocity scale. At finite Reynolds numbers the power law exponent is not universal and may be dependent not only of Reynolds number but of the flow type.

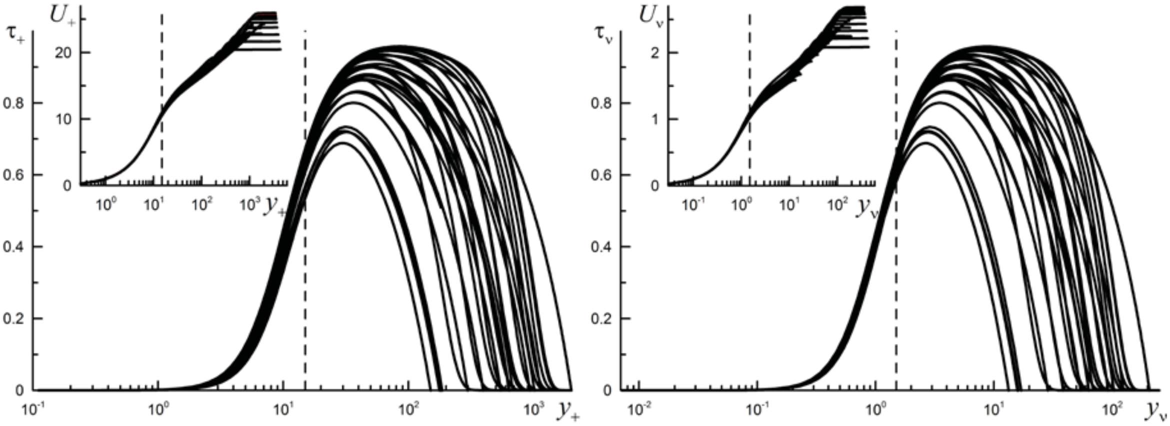

Fig. 1. Comparison of classical (left) and new (right) scaling of Reynolds shear stresses and mean velocity (inserts). Curves – DNS data in the range $180 < \mathrm{Re}_\tau < 2000$, dashed lines mark off the rough border of viscous sublayer.

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References

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