Non-equilibrium quantum dynamics of strongly correlated ultracold atoms in optical lattices.

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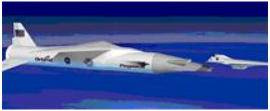
Goal:

Create many-body systems with interesting collective properties

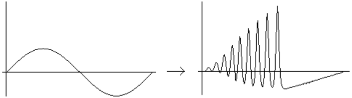
Keep them simple enough to be able to control and understand them



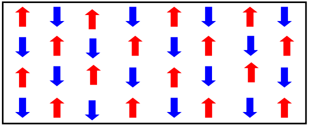
Universaility in dynamics of nonlinear classical systems

Universality in quantum many-body systems in equilibrium

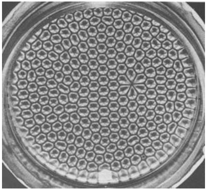
Solitons in nonlinear wave propagation



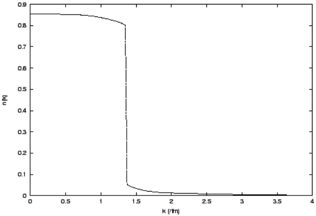
Broken symmetries



Bernard cells in the presence of T gradient



Fermi liquid state



Do we have universality in nonequilibrium dynamics of many-body quantum systems?

Why study dynamics in quantum many-body systems of ultracold atoms

Long intrinsic time scales
- Interaction energy and bandwidth ~ 1 kHz
- System parameters can be changed over this time scale

Decoupling from external environment
- Long coherence times

Can achieve highly non equilibrium quantum many-body states

$$H_i \rightarrow H_f \qquad\qquad |\Psi(t)\rangle = e^{-iH_f t}|\Psi_i\rangle$$

Rich toolbox for probing transient many-body states

This talk:

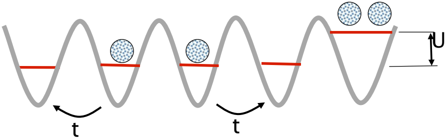
Nonequilibrium dynamics of ultracold bosons in an optical lattice

Density relaxation results in formation of oscillatory zones

Appearance of semiclassical solitons

Nature of solitons: KdV-like

Bose Hubbard model



$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1)$$

Hard core limit $U \to \infty$

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \hat{P}\, b_i^\dagger\, b_j\, \hat{P}$$

$\hat{P}$ – projector of no multiple occupancies

Spin representation of the hard core bosons Hamiltonian

$$\mathcal{H} = -t \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y)$$

OUTLINE

The behavior of the corresponding quantum dynamic depends strongly on the parameters (μ_0 and $J_z/J_\perp$). In general, we can observe:

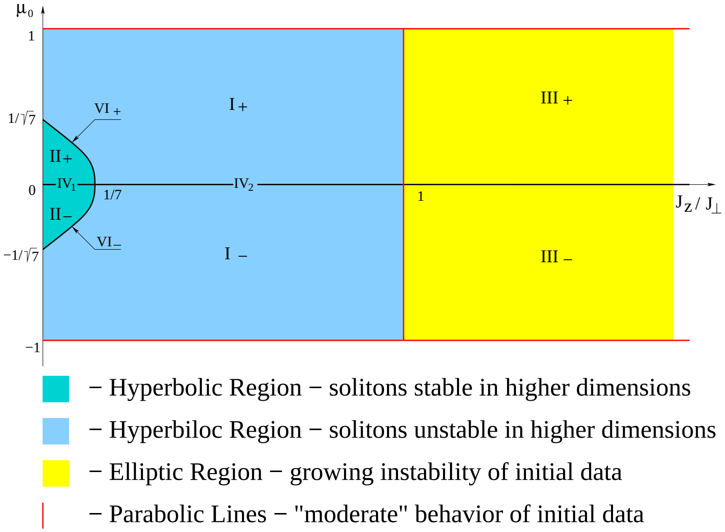
1) Particle-type solitons;

2) Hole-type solitons;

3) Modulation instabilities of elliptic type with the appearance of oscillation zone;

4) Non-trivial two-dimensional "lump" solutions.

The different regimes of the dynamics of the pattern of the cold atoms can be represented on the phase diagram (Fig. 8).



Two main regions can be marked on the Phase Diagram:

1) Hyperbolic Region;

2) Elliptic region.

In the Hyperbolic Region any initial data are separated into the left-moving and the right-moving parts with the separate evolution of the left-moving and the right-moving part.

In the Elliptic Region initial data demonstrate growing elliptic instability with the appearance of the oscillation zone at the final stage.

Parabolic Lines - appear at the boundaries of the Hyperbolic and the Elliptic Regions - solutions do not break into the left- and the right-moving parts - elliptic effects are "moderate".

The whole diagram can be separated into general parts $\mu > 0$ ($n > 1/2$) and $\mu < 0$ ($n < 1/2$) according to the particle-hole symmetry.

HYPERBOLIC REGION

The Hyperbolic Region can be separated into two important parts:

1) Part I - corresponding to appearance of soliton solutions unstable in the higher dimensions and presence of rather nontrivial "lump" solutions in two-dimensions;

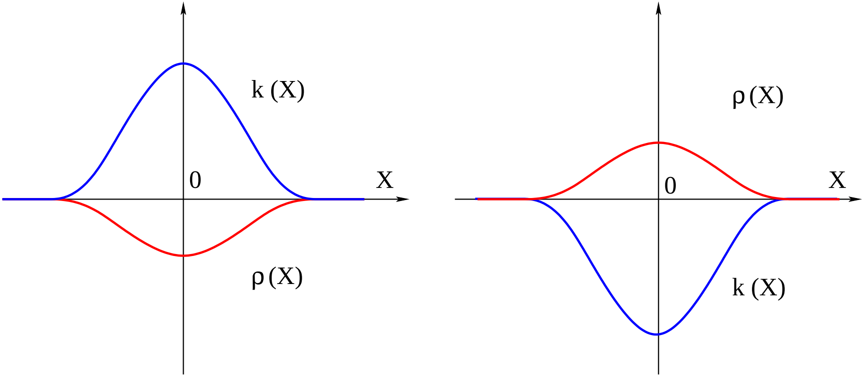
2) Part II - corresponding to appearance of soliton solutions stable in the higher dimensions;

Part I and Part II are separated by the line corresponding to appearance of short-period oscillating solutions.

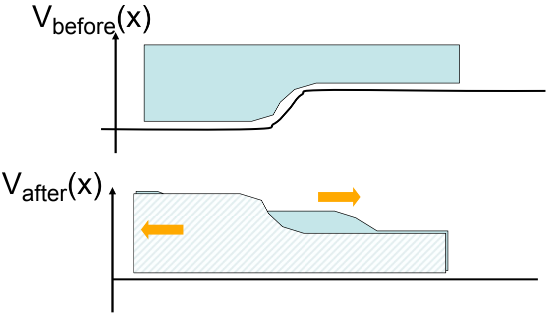
Consider the upper half $\mu > 0$ ($n > 1/2$):

The soliton solutions unstable in the higher dimensions are the particle-type soliton solutions (Region I_+) (Fig. 1);

The soliton solutions stable in the higher dimensions are the hole-type soliton solutions (Region II_+) (Fig. 1).

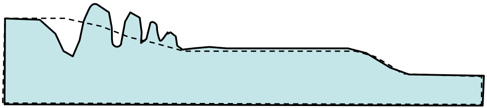


Equilibration of density inhomogeneity



Suddenly change the potential. Observe density redistribution

Strongly correlated atoms in an optical lattice: appearance of oscillation zone on one of the edges



Semiclassical dynamics of bosons in optical lattice: Kortweg- de Vries equation

Instabilities to transverse modulation

For the analysis of the qualitative behavior of the quantum dynamics the evolution of the "step-like" initial data can be very useful.

Zone I_+.

Typical evolution of the "step-like" initial data of a small amplitude for the zone I_+ is represented at Fig. 2 - 6.

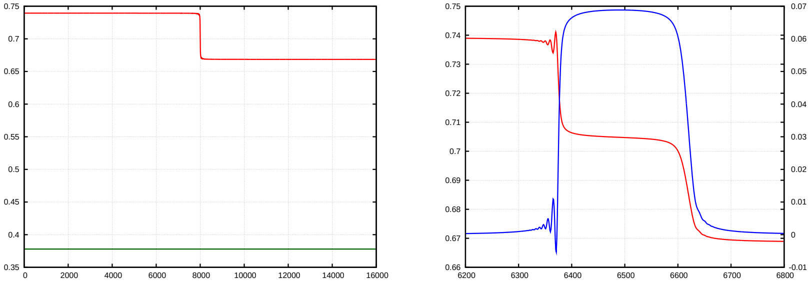


Figure: The separation of the left- and the right-moving parts with the oscillations formation in the left-moving part for step-like initial conditions of rather small amplitude for the Zone I_+. (The red and the blue lines correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

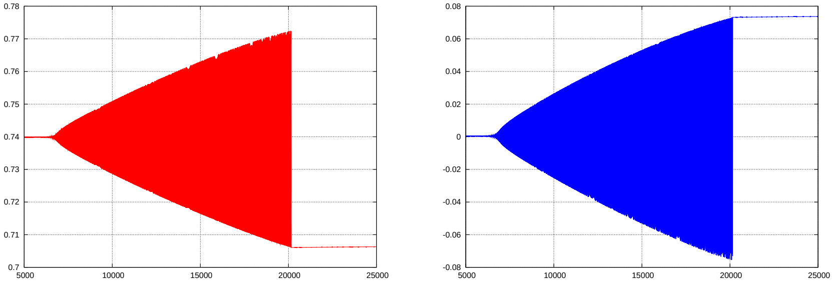


Figure: The form of the oscillation zone (t=16000) in the left-moving part for the Zone I_+. (The left and the right pictures correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

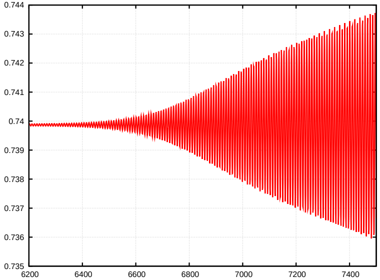


Figure: Generating of the oscillations of a small amplitude at the trailing edge of the oscillation zone in the left-moving part for the Zone I_+. (The left and the right pictures correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

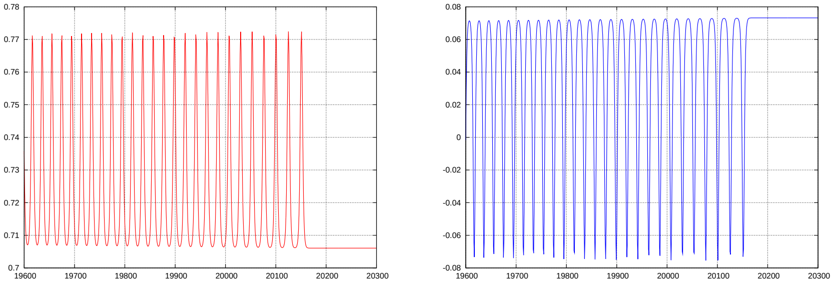


Figure: Formation of free solitons at the leading edge of the oscillation zone in the left-moving part for the Zone I_+. (The left and the right pictures correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

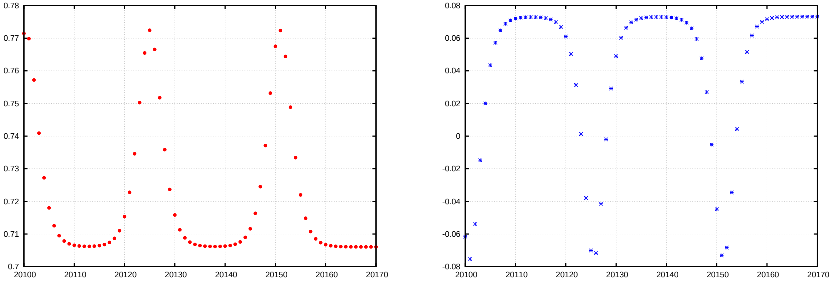


Figure: The form of discrete solitons at the leading edge of the oscillation zone in the left-moving part for the Zone I_+. (The left and the right pictures correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

More interesting features arise after the increasing of the amplitude of the step-like initial conditions.

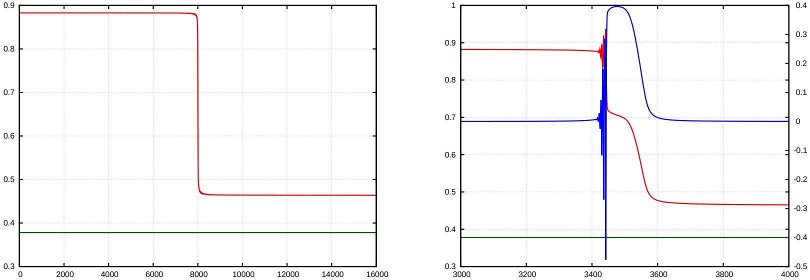


Figure: The separation of the left- and the right-moving parts with the oscillations formation for the step-like initial conditions of rather big amplitude. (The green line marks the value $\mu_0 = 1/\sqrt{7}$. The red and the the blue lines correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

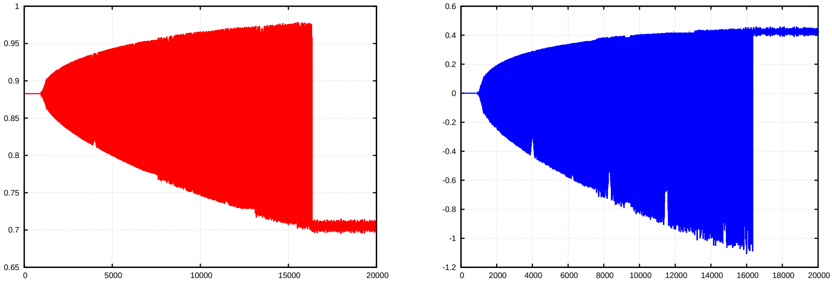


Figure: The form of the oscillation zone (t = 5000) in the left-moving part for the step-like initial conditions of rather big amplitude. (The left and the right pictures correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

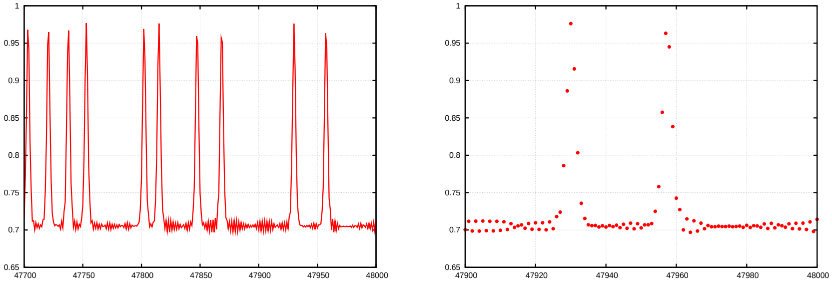


Figure: Solitons in the variables $\mu_i(t)$ at the leading edge of the oscillation zone in the left-moving part for the step-like initial conditions of rather big amplitude. The last picture represents the discrete form of the solitons at the leading edge ($t = 15000$).

The interaction of the solitons of big amplitude demonstrate interesting tendency to "pairing".

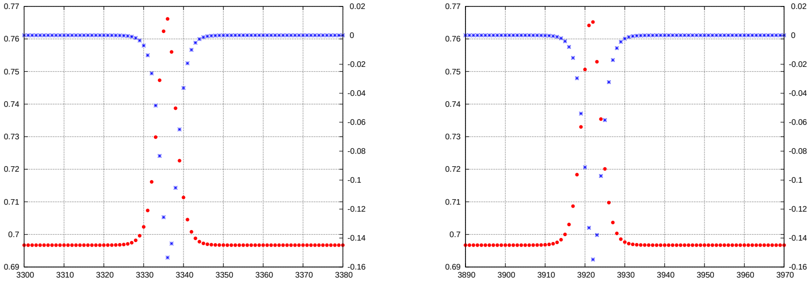


Figure: The propagation of a soliton solution for the Region II_+ in the (μ_i, k_i)-variables for rather big times t ($k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$). Here $t_1 = 20000$, $t_2 = 38000$.

Zone II_+.

Let us consider now the zone II_+ where the solitons of the hole type are expected for $\mu_0 > 0$. We represent the evolution of the step-like initial data at Fig. 11 - 13.

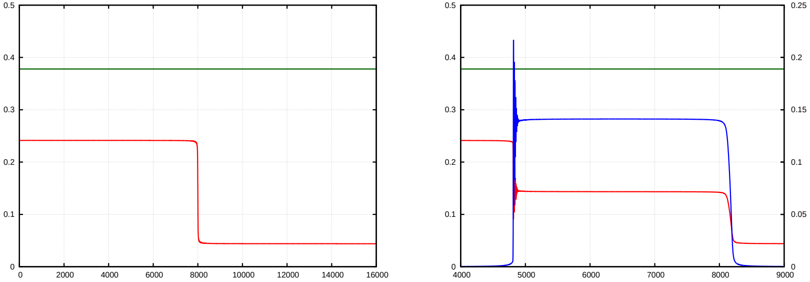


Figure: The separation of the left- and the right-moving parts with the oscillations formation in the left-moving part for step-like initial conditions for Zone II_+. (The green line marks the value $\mu_0 = 1/\sqrt{7}$. The red and the the blue lines correspond to the variables $\mu_i(t)$ and $k_i(t) = \varphi_{i+1}(t) - \varphi_i(t)$ respectively).

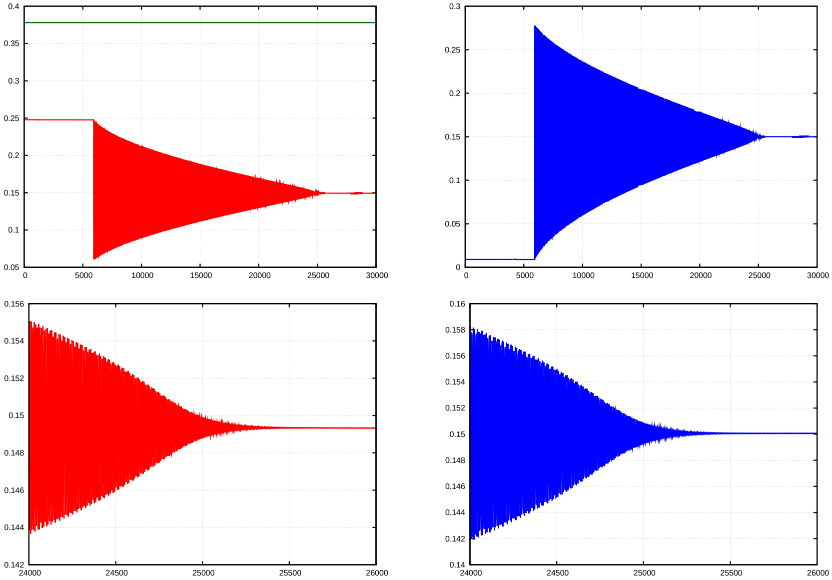


Figure: The form of the oscillation zone (t = 60000) and the generating of the oscillations of a small amplitude at the trailing edge of the oscillation zone in the

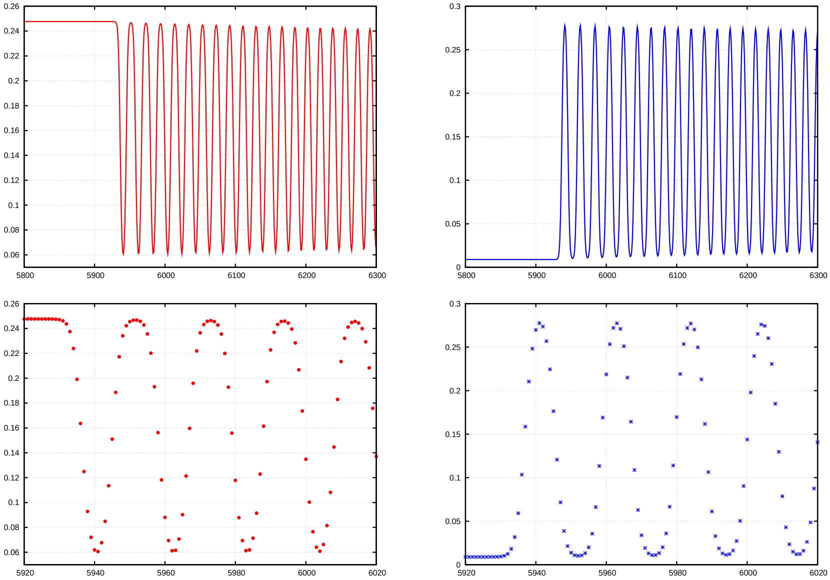


Figure: Formation of free solitons at the leading edge and the form of discrete solitons at the leading edge of the oscillation zone in the left-moving part for the

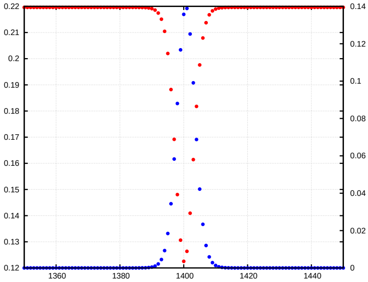
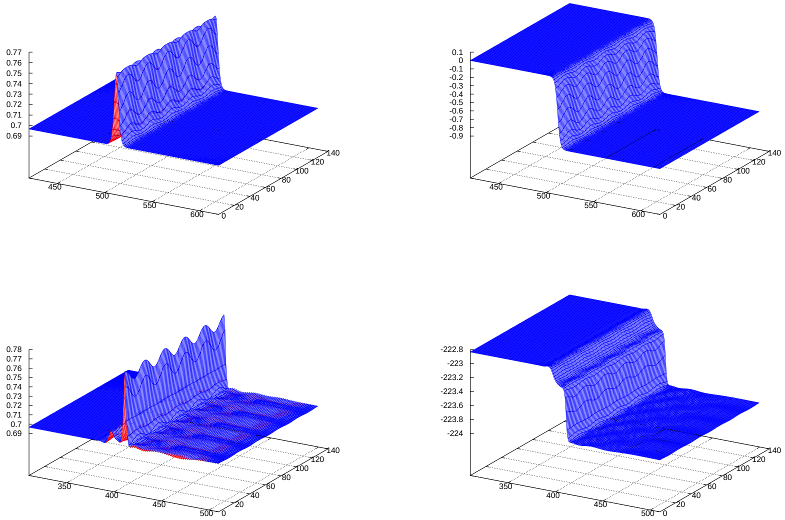


Figure: The propagation of a soliton solution for the Region II_+ in the (μ_i, k_i)-variables for rather big times t $(k_i(t) = \varphi_{i+1}(t) - \varphi_i(t))$. Here $t_1 = 10000,\ t_2 = 23000$.

Two-dimensional modulations. Zone I_+.



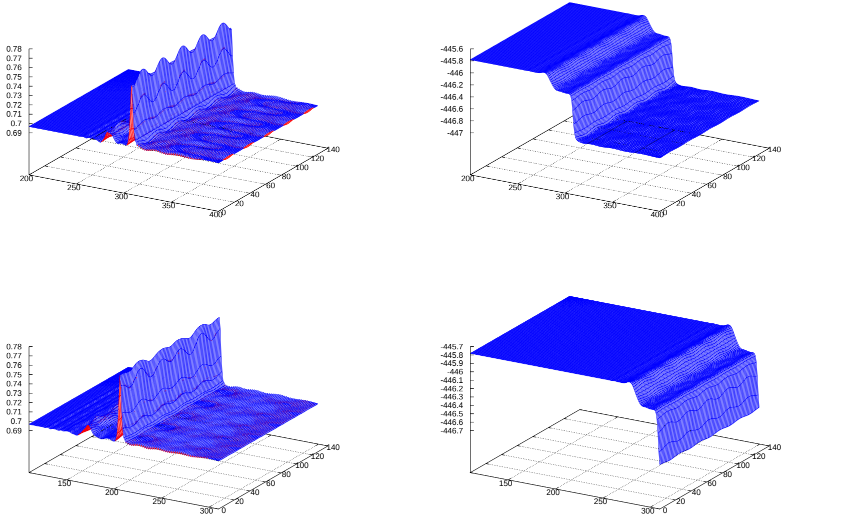


Figure: The evolution of the slightly modulated soliton in two dimensions in the (μ, φ)-variables for the Region I$_+$. Here $t_0 = 40$, $t_1 = 60$.

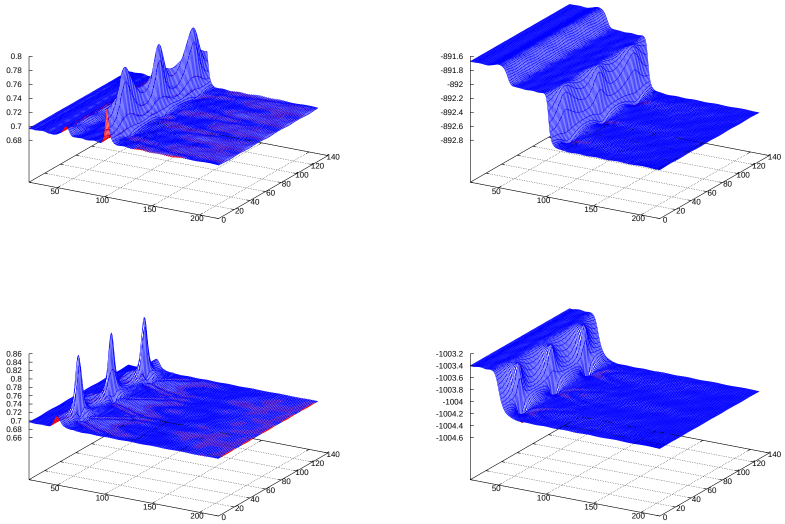
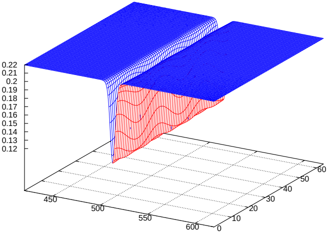


Figure: The evolution of the slightly modulated soliton in two dimensions in the (μ, φ)-variables for the Region I_+. Here $t_5 = 80$, $t_6 = 90$. Appearance of the

Two-dimensional modulations. Zone II_+.



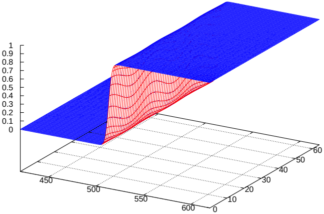


Figure: The initial conditions in the form of slightly modulated soliton in the (μ, φ)-variables for the Region II_+.

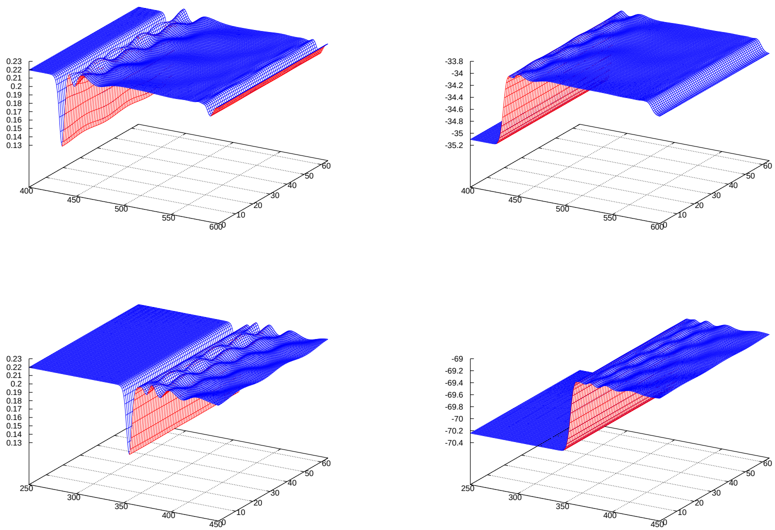


Figure: The evolution of the slightly modulated soliton in two dimensions in the (u, φ)-variables for the Region II_+. Here $t_0 = 10$, $t_2 = 20$

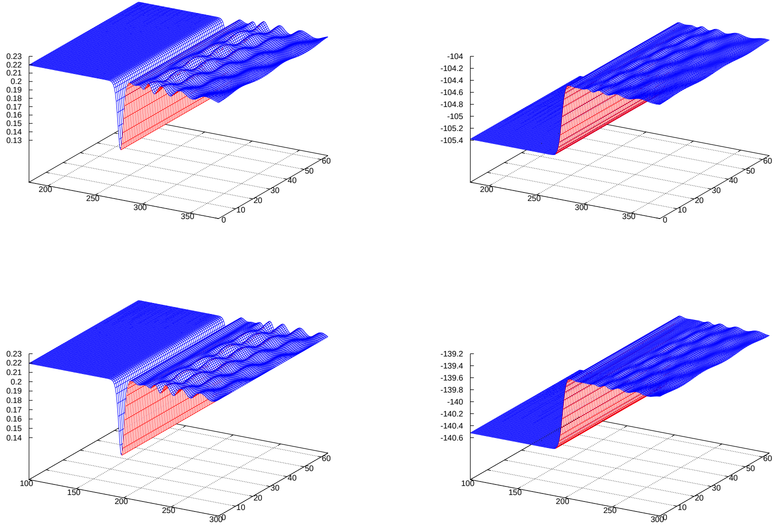


Figure: The evolution of the slightly modulated soliton in two dimensions in the (μ, φ)-variables for the Region II. Here $t_4 = 30$, $t_5 = 40$.

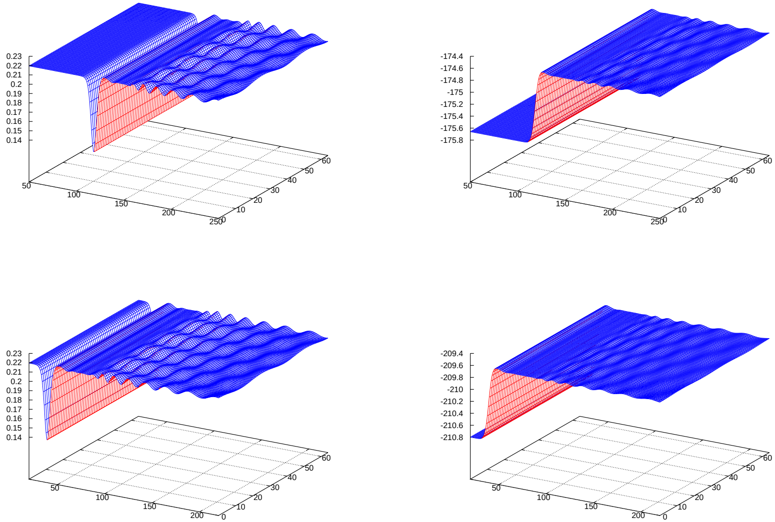


Figure: The evolution of the slightly modulated soliton in two dimensions in the (u, φ)-variables for the Region II. Here $t_c = 50$, $t_z = 60$.

Soliton dynamics in the parabolic potential.

Consider the experimental situation when the lattice of the cold atoms is placed in some parabolic potential. The parabolic potential is assumed rather slow with respect to the lattice constant such it can be approximated locally just by local chemical potential slowly varying in space. The corresponding equilibrium density distribution corresponding to the constant phase ($\varphi_i = const$) can be calculated in the long-wave limit using the hydrodynamic part of the lagrangian.

We investigate the soliton propagation on the parabolic potential. The following initial conditions can be considered in this situation:

1) The soliton has a particle type and belongs to the zone I_+ of the diagram 8;

2) The soliton has a hole type and belongs to the zone II_+ of the diagram 8;

3) The soliton has a particle type and belongs to the zone II_- of the diagram 8;

4) The soliton has a hole type and belongs to the zone I_- of the diagram 8.

Soliton solutions propagate without essential change while being at the same zone of the Phase Diagram 8 and collapse during transitions between different zones of Diagram 8.

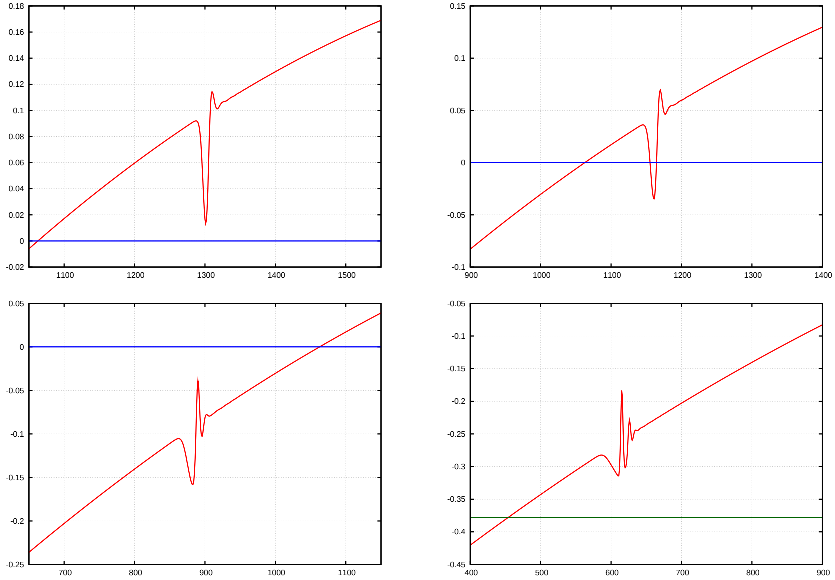


Figure: The destruction of the left-moving hole-type soliton near the value $\mu_0 = 0$ (blue line) in a parabolic potential ($U_+ \to U_-$). Here $t_1 = 125$, $t_2 = 150$

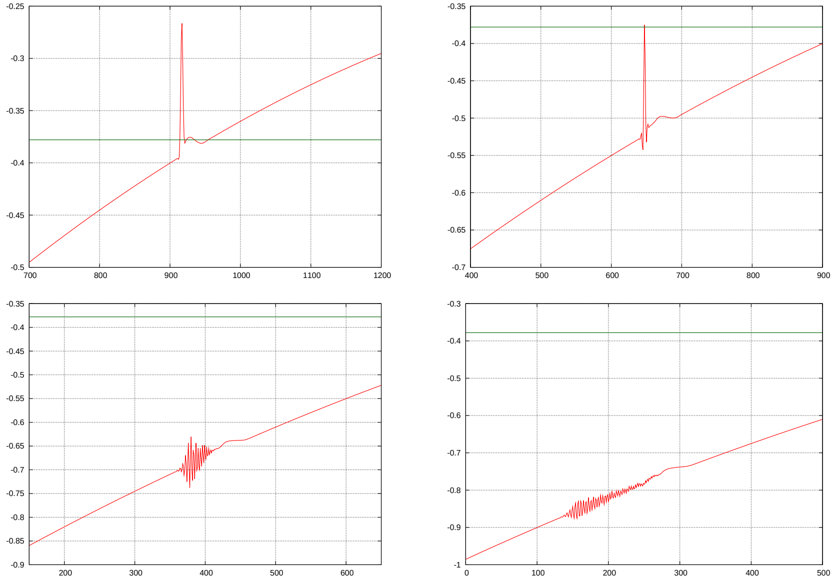


Figure: The destruction of the left-moving particle-type soliton near the value $u_0 = -1/\sqrt{7}$ (green line) on the parabolic potential. (I... $\to$ I...). Here $t_1 = 150$

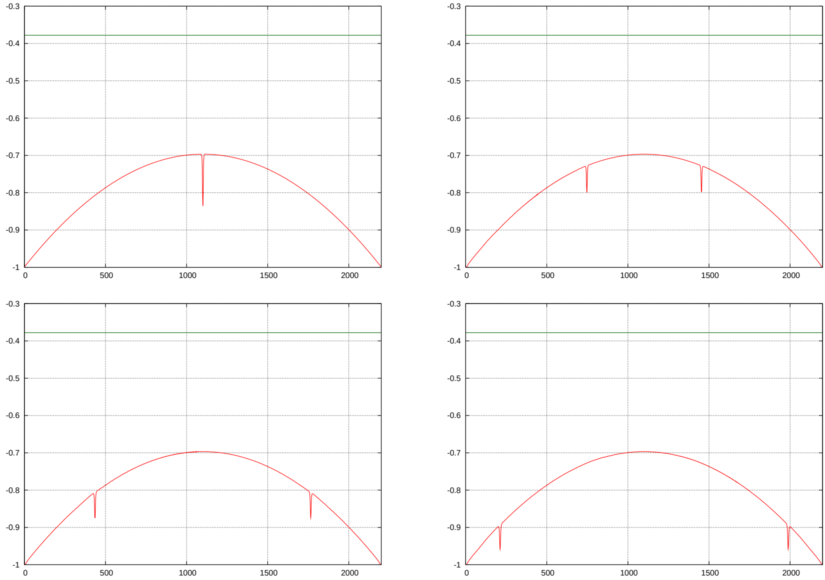
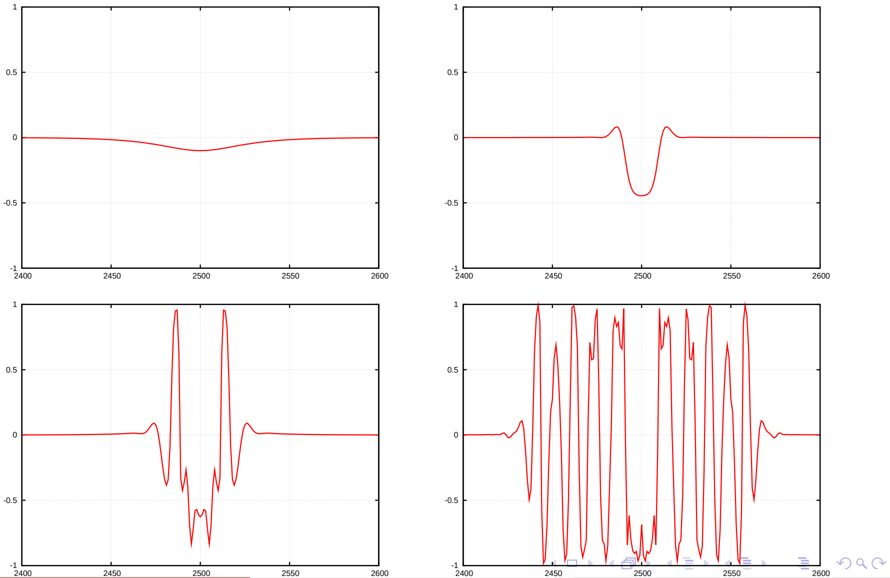
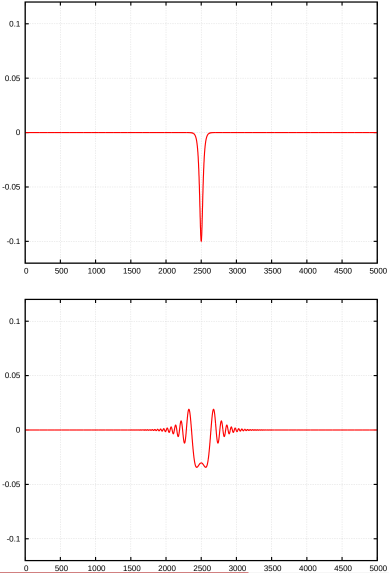


Figure: The propagation of the hole-type solitons of the zone I_- on the slow parabolic equilibrium density distribution. Here $t_1 = 0$, $t_2 = 100$, $t_3 = 200$

Elliptic Region



Parabolic Line



Conclusions

Patterns of cold atoms can demonstrate rather nontrivial dynamics on the long time scales. Depending on the values of parameters different solutions like

- particle-type solitons

- hole-type solitons

- solitons stable in the higher dimensions

- solitons unstable in the higher dimensions

- interacting solitons

- two-dimensional "lump" solutions

- modulation instability

can be observed.