Rogue waves and intermediate shock instability in Alfvenic turbulence

P.L. Sulem
Université de Nice-Sophia Antipolis, CNRS,
Observatoire de la Côte d'Azur, Nice, France

Collaborations: T. Passot, D. Laveder, G. Sanchez-Arriaga

SCT2012, Novosibirsk, June 4-8, 2012

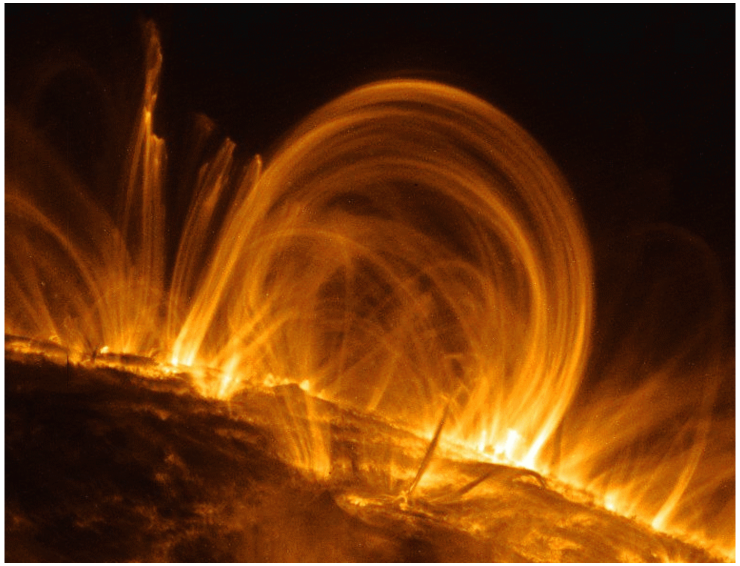
Research supported in part by the European Commission's 7th Framework Program (PF7:2007-2013) under grant agreement SHOCK (project # 284515) and Programme Terre-Soleil of INSU-CNRS.

SOLAR AND HELIOSPHERIC COLLISIONLESS KINETICS

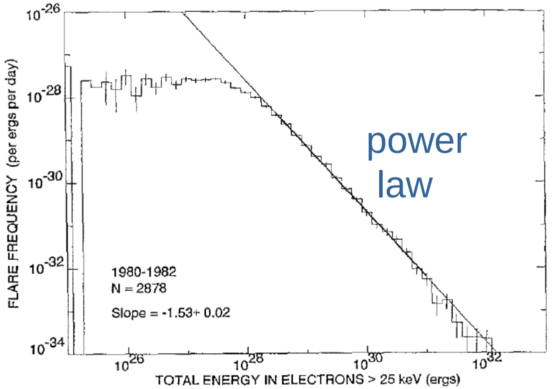
Alfvén waves are ubiquitous in space and astrophysical plasmas
(mostly because only weakly dissipated and can thus propagate on long distances).

Possibly contribute to the heating of solar corona.

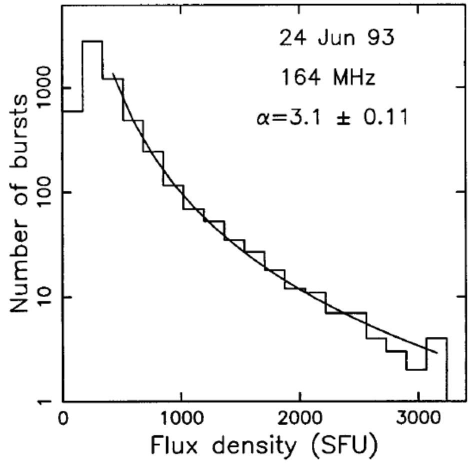
Sun: magnetic structures



Very intermittent structures: myriad of coronal loops;
permanent **impulsive** reorganization
of magnetic structures.



Distribution of flares
(observations in hard X-rays)
Crosby et al.
Solar Phys. ***143****, 275 (1993)*



At much smaller energies (10^{21} ergs) the power law has a much steeper slope.

Merciet and Trotte,
ApJ ***474****, L65 (1997)*

Coronal nanoflares are related to **intermittent dissipative events** in MHD turbulence and might contribute to coronal heating.

Question: Role played by Alfvén waves in these phenomena?

Requires progress in the understanding of Alfvén wave dynamics.

Outline

1) Model equations for nonlinear Alfvén waves:
- Derivative nonlinear Schrödinger equation for ***dispersive*** Alfvén waves
- Cohen-Kulsrud equation for ***non-dispersive*** Alfvén waves: a paradigm of non strictly hyperbolic system.

2) Non-dispersive regime
Viscous instability and (quasi) gradient-collapse of intermediate shocks,
Reconnection and polarization reversal (associated with *phase-jump reduction*).
Time-intermittency of dissipation in the turbulent regime.

3) Influence of dispersion
Competition between dissipation and dispersion.
Formation of dark or (oblique) bright solitons,
Reconnection and/or amplitude collapse: *phase jump reduction*.
Formation of **huge** Alfvenic rogue waves in the turbulent regime.

4) Conclusion
In all these regimes, *breaking of a topological constraint* through violent events, needed for significant dissipation.

The model equations

Starting point: Hall-MHD equations

$$\partial_t \rho + \nabla.(\rho \mathbf{u}) = 0$$

$$\rho(\partial_t \mathbf{u} + \mathbf{u}.\nabla \mathbf{u}) = -\frac{\beta}{\gamma}\nabla \rho^\gamma + (\nabla \times \mathbf{b}) \times \mathbf{b}$$

$$\partial_t \mathbf{b} - \nabla \times (\mathbf{u} \times \mathbf{b}) = \underbrace{-\frac{1}{R_i}\nabla \times (\frac{1}{\rho}(\nabla \times \mathbf{b}) \times \mathbf{b})}_{\text{Hall term}}$$

$$\nabla.\mathbf{b} = 0$$

velocity unit: Alfvén speed
length unit : $R_i \times$ ion inertial length
time unit: $R_i \times$ ion gyroperiod
density unit: mean density
magnetic field unit: ambient field

Originates from writing Ohm's law as: $E=-v_e xB$
Accounts for electron-ion decoupling

In the presence of an ambient field, the Hall term induces **dispersive effects.**

Focus on the **Alfvén wave** dynamics can be made by using a long-wavelength reductive perturbative expansion (assume small-amplitude weakly dispersive waves).

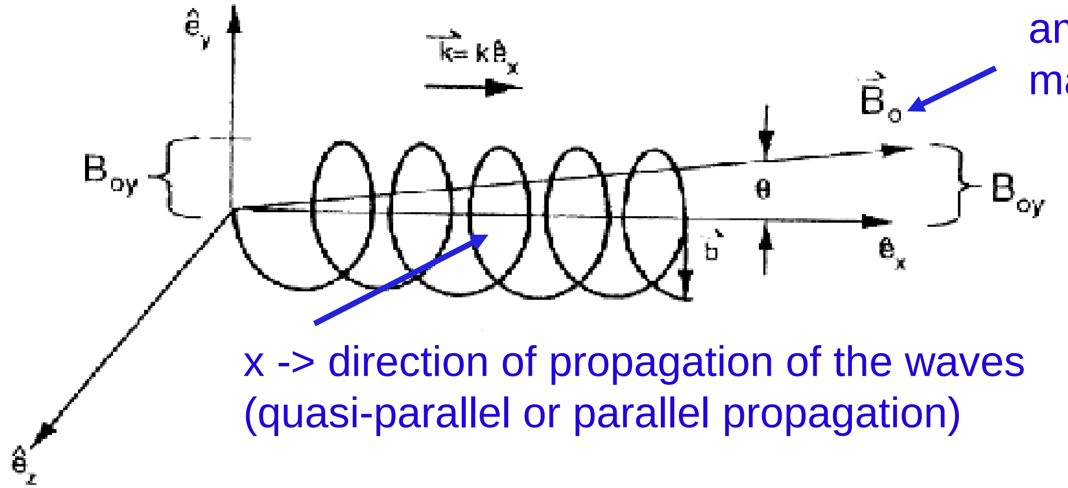
For waves propagating parallel or quasi-parallel to the ambient field:

Reductive perturbative expansion ⇒ derivative nonlinear Schrödinger equation (DNLS).

DNLS equation also valid in any oblique direction in the large β limit (Ruderman, JPP **67**, 271, 2002)

$$\mathrm{b} \equiv b_y + \imath b_z + B_{0y}$$

transverse component of the ambient field
(included in the initial conditions)



ambient field B_0
making an angle with x

x -> direction of propagation of the waves
(quasi-parallel or parallel propagation)

- Weakly nonlinear, weakly dispersive regime:

$$\mathbf{b}_\perp, \mathbf{u}_\perp \sim \varepsilon^{1/2}, \quad x \sim \varepsilon, \quad t \sim \varepsilon^2$$

- Slaved sonic waves :

$$\rho \sim |\mathbf{b}_\perp|^2$$

Derivative nonlinear Schrödinger(DNLS) equation

$$\partial_t b + \alpha \partial_x (|b|^2 - \underbrace{\langle |b|^2 \rangle}_{\text{Instantaneous space average}}) b] + i\delta \partial_{xx} b = 0$$

$$x = \pm\infty, b = 0 \text{ if parallel propagation}$$
$$x = \pm\infty, b \neq 0 \text{ if slightly oblique propagation}$$

Integrable by Inverse Scattering Transform
(Kaup& Newell 1978, Gerdzhikov et al. 1980, Kawata & Inoue 1978, Chen & Lam 2004) for both zero BC (parallel propagation) and non zero BC (weakly oblique propagation).

Retaining dissipation processes

In a collisionless plasma : Landau damping

σ is a small positive parameter
that depends on the particle distributions and
vanishes in the limit of cold plasmas

$$\partial_t b + \partial_x[(\alpha - \sigma\mathcal{H})(|b|^2 - \langle|b|^2\rangle)b] + i\delta\partial_{xx}b = 0$$

$$\mathcal{H}(|b|^2) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{+\infty}\frac{|b|^2(x')}{x'-x}dx',$$

Kinetic DNLS (KDNLS) equation

derived from Vlasov-Maxwell equations
(Rogister 1971, Mjolhus & Wyller 1988, Passot & Sulem PoP 2003)

In a collisional plasma: viscosity and magnetic diffusivity

$$\partial_t b + \alpha\partial_x(|b|^2 - \langle|b|^2\rangle)b] + i\delta\partial_{xx}b - \eta\partial_{xx}b = 0$$

When viscosity but no dispersion (δ=0): Cohen-Kulsrud equation.
A prototype of non strictly hyperbolic system.

The non dispersive regime: Cohen Kulsrud equation

$$\boxed{\partial_t b + \alpha\partial_x[(|b|^2 - \langle|b|^2\rangle)b] = \eta\partial_{xx}b}$$

α= 1/[4(1−β)]; In the following, β< 1 and thus α> 0.

Paradigm for non strictly hyperbolic systems.
Rankine-Hugoniot (RH) conditions do not uniquely specify shock dynamics.
Zero-viscosity limit is not necessarily well defined.

$$b = |b|e^{i\theta}$$

- **With no dissipation:** two sets of RH jump conditions:

  One connects two states in which only the phase θ changes (rotational discontinuities).

  The other connects two among three states in such a way that either only the amplitude |b| changes (fast shocks) or both |b| and θ change (intermediate shocks, that only exists for a phase jump |Δθ|=π).

- **With viscosity**, intermediate and fast shocks persist and transform into structures characterized by a finite width, while rotational discontinuities cannot exist anymore.

  Finite-dissipation intermediate shocks with a phase jump |Δθ|=π (the only one that can propagate without distortion) are not uniquely specified by the RH conditions.

  Intermediate shocks with |Δθ| ≠π change shape.

Previous studies mostly concern Riemann problem
(*Kennel et al. 1990, Wu & Kennel, 1992, 1993, Wu 2003*).

$$b = b_y + ib_z \qquad b = |b|e^{i\theta}$$

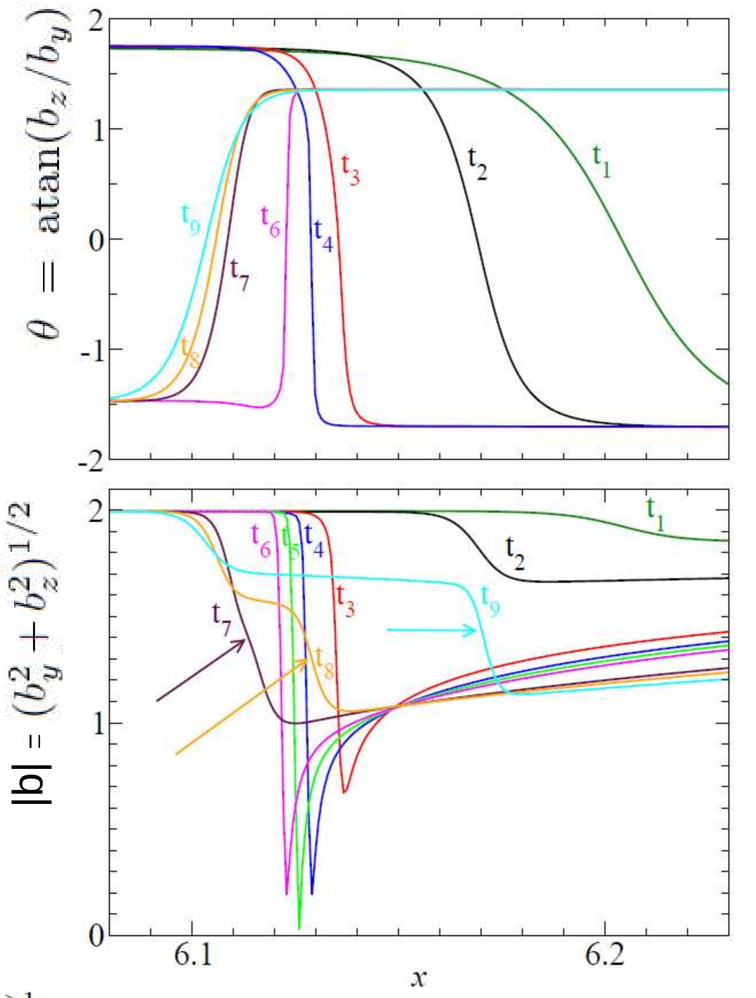
$$\begin{cases} \partial_t|b| + \alpha(3|b|^2 - \langle|b|^2\rangle)\partial_x|b| = \eta\left[\partial_{xx}|b| - |b|(\partial_x\theta)^2\right] \\ \partial_t\theta + \alpha(|b|^2 - \langle|b|^2\rangle)\partial_x\theta = \eta\left[\partial_{xx}\theta + \frac{2}{|b|}\partial_x|b|\partial_x\theta\right]. \end{cases}$$

►Intermediate shocks with angular jump $|\Delta\theta| < \pi$ broaden.

►Intermediate shocks with $|\Delta\theta| > \pi$ are unstable:
front steepens and amplitude jump increases up to

- the formation of a neutral point for the b field (reconnection)
- a change in the wave polarization **with a phase jump reduced by 2π**.
- A fast shock is simultaneously emitted, and the intermediate shock, now with $|\Delta\theta| < \pi$, slowly dissipates.

Evolution from a rotational discontinuity



Arrows indicate emitted fast shocks

At least not too close to the collapse time, the amplitude is slaved to the phase (supported by numerics):

$$\begin{cases} \alpha\partial_x\alpha(|b|^2 - \langle|b|^2\rangle) = \eta(\partial_x\theta)^2 \\ \partial_t\theta + \alpha(|b|^2 - b_0^2)\partial_x\theta = \eta\partial_{xx}\theta \end{cases}$$

Within the shock,

$$|b| = b_0(t)\left(1 + \frac{\eta}{b_0^2(t)l(t)}\widetilde{b}(\xi)\right) \qquad \xi = x/l(t)$$

$$\theta = \theta(\xi) \qquad l(t) = [l^2(0)+\lambda\eta t]^{1/2}$$

$$\theta = \pm A\left(\frac{\pi}{2} + \sin\tanh(\xi)\right) \qquad l(t) = \sqrt{l_0^2 + \frac{4}{3}(1-A^2)t}$$

Instability when A>1

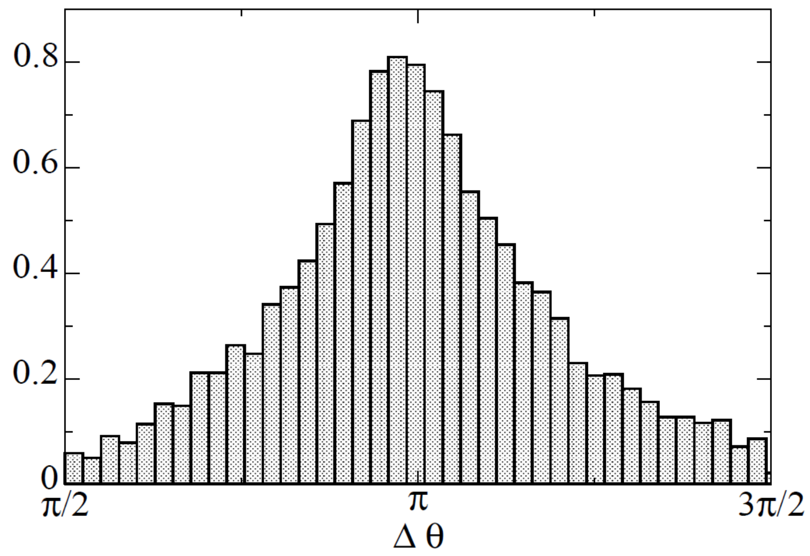
The turbulent regime

(when **a large-scale random driving is supplemented to the CK equation)**

Phase jumps without appreciable amplitude variations arise under the action of the forcing.

These structures which are in fact intermediate shocks
propagate and either diffuse (if $|\Delta\theta| < \pi$) or steepen (if $|\Delta\theta| > \pi$)
on a timescale $O(1/\eta)$, at least far enough from reconnection.

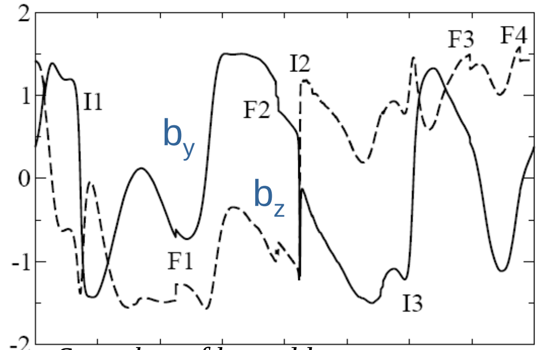
Evolution time scale increases
as $|\Delta\theta|$ approches π, which
favors phase jumps of order π



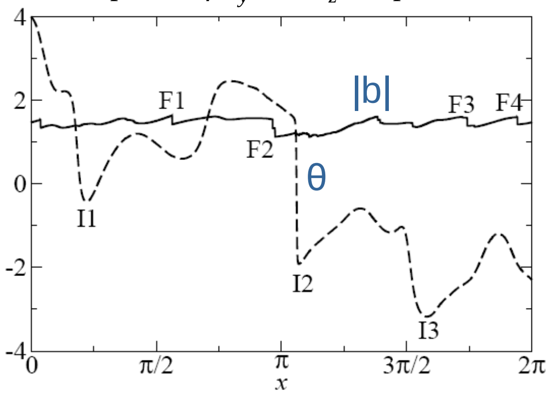
Probability distribution of phase jumps
in a turbulence simulation.

Intermediate-shock instability also arises in turbulent regime

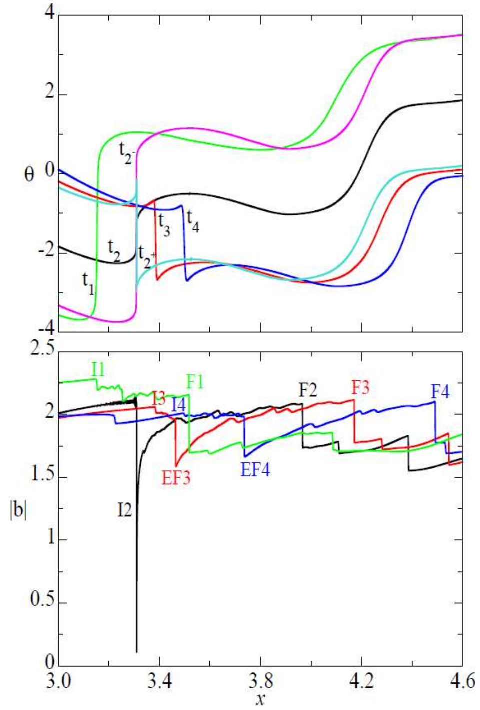
$\cdot\ \eta = 10^{-4}$



Snapshot of b_y and b_z components

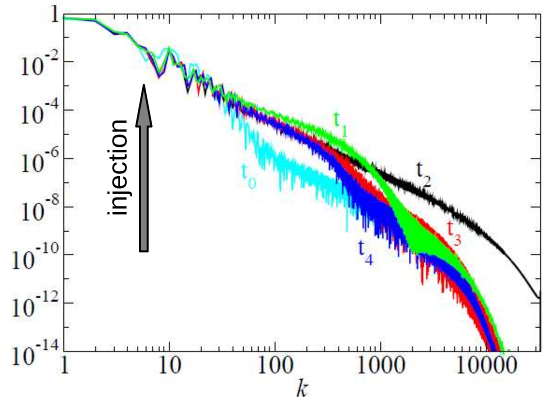


Snapshot of the amplitude (solid line) and phase (dash line)

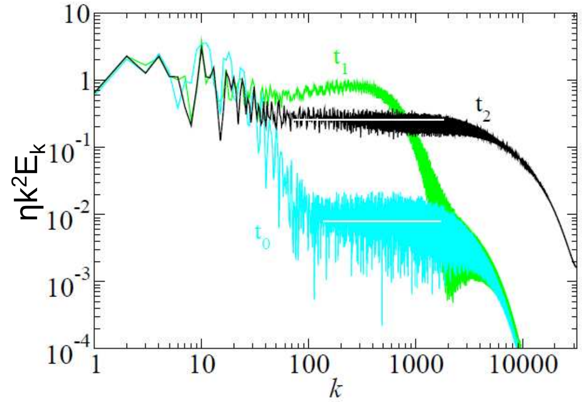


Intermediate-shock instability:
Very similar to the deterministic case:

Phase reverses near time t_2
(phase jump reduced by 2π)
Reconnection
Emission of a fast shock



Energy spectrum



Two dissipation zones, associated with intermediate and fast shocks (except at the reconnection time).

F: fast shock
I: Intermediate shock
EF: emitted fast shock

Energy dissipation

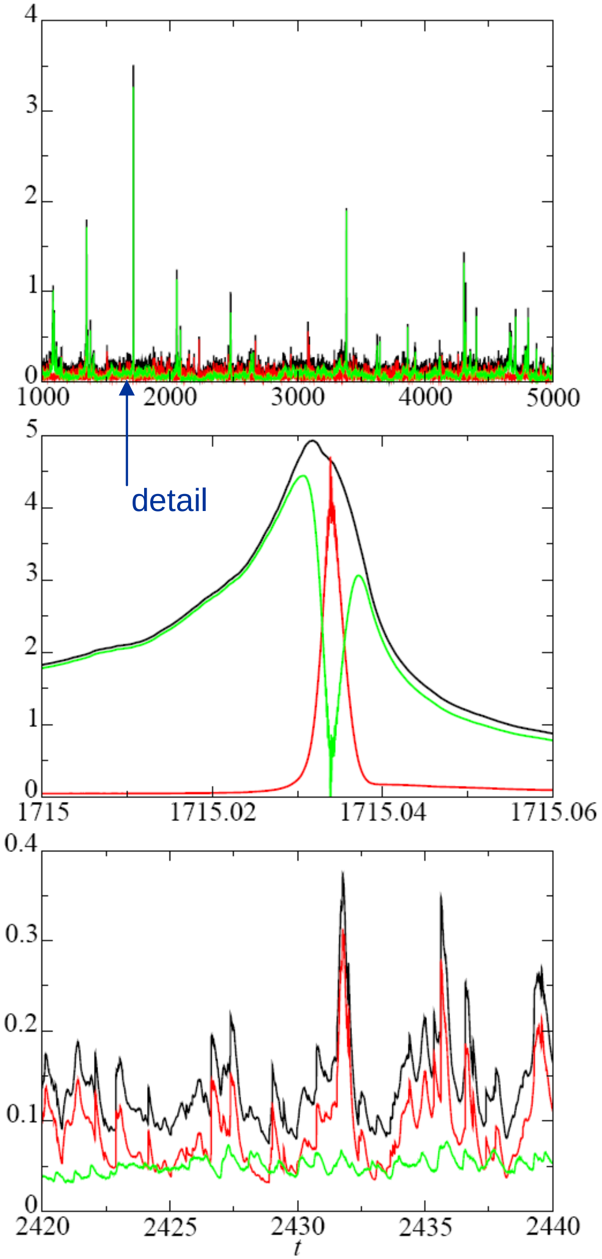
The dissipation $D(t) = 2\eta \int |\partial_x b|^2 dx$ can be separated in two parts:

$$\underline{D(t)} = D_\theta(t) + D_B(t)$$
$$= \eta \underbrace{\int |b|^2 (\partial_x \theta)^2 dx}_{} + \eta \underbrace{\int (\partial_x |b|)^2 dx}_{}.$$

The total dissipation D(t) is strongly intermittent: background + isolated intense peaks associated with instabilities of intermediate shocks.

Total dissipation D (black) and its components D_B (red) and D_θ (green).

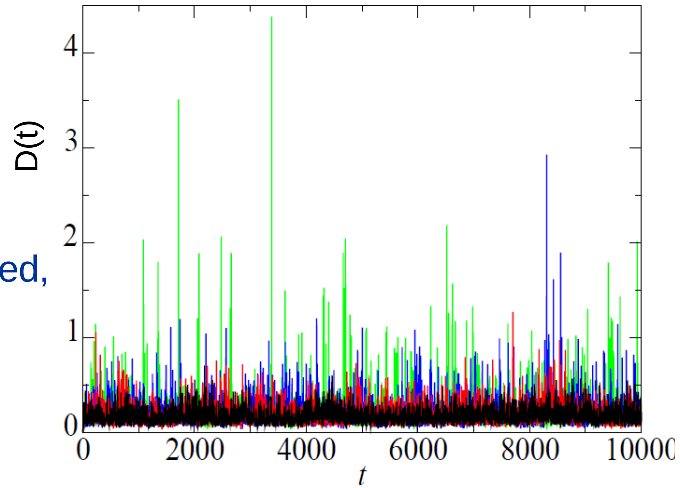
Bottom panel: between dissipation bursts, dissipation is mostly due to fast shocks.



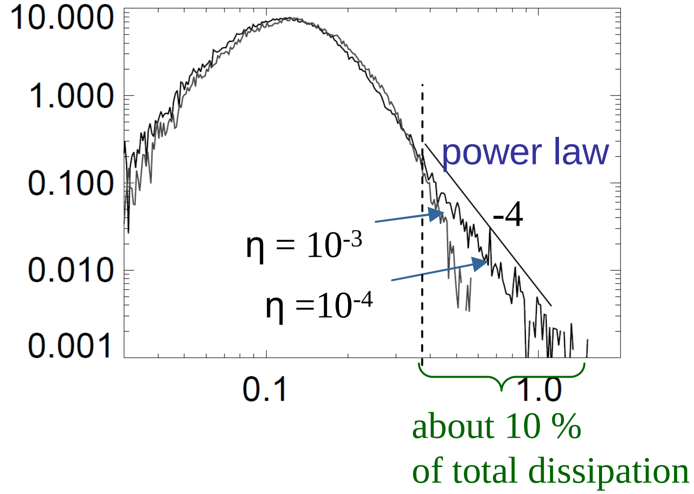
Total dissipation for various viscosities

$\eta = 10^{-3}$
$\eta = 4.\ 10^{-4}$
$\eta = 2.\ 10^{-4}$
$\eta = 1.\ 10^{-4}$

When viscosity is decreased, dissipation bursts are less frequent but more intense.

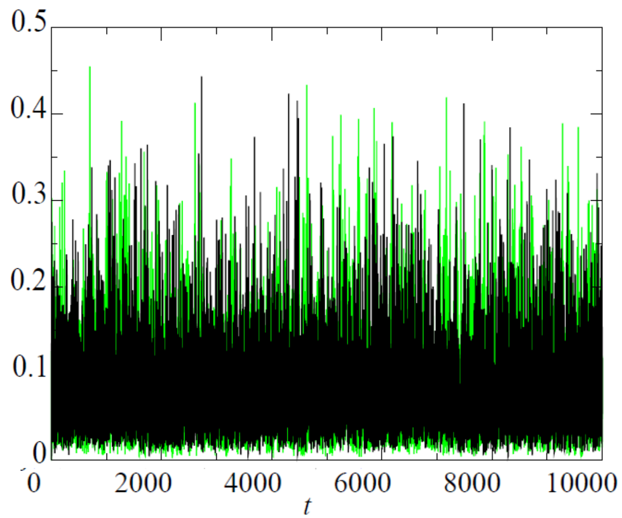


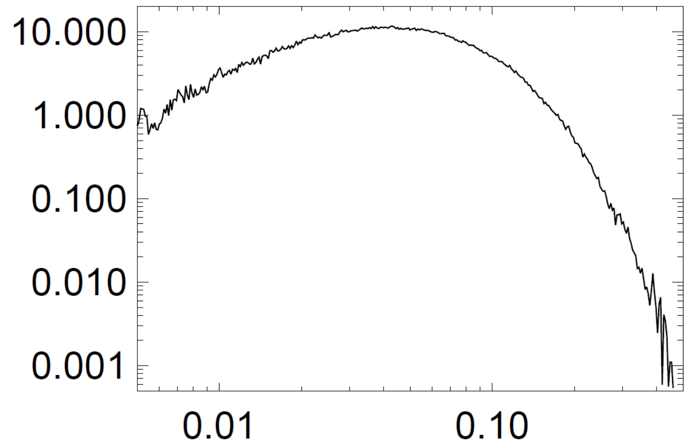
Histograms of the dissipation



For comparison: **Burgers equation,**

well defined limit
as $\nu \rightarrow 0$





Energy $\mathcal{E}(t) = \int |b(x,t)|^2 dx$

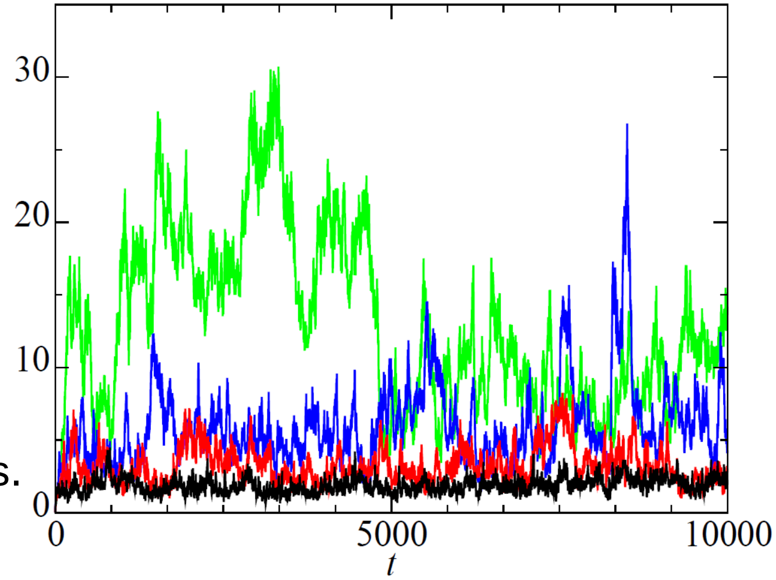
$\eta = 10^{-3}$

$\eta = 4.\ 10^{-4}$

$\eta = 2.\ 10^{-4}$

$\eta = 10^{-4}$

Decreasing η, strongly enhances energy fluctuations.



Energy is sensitive to viscosity

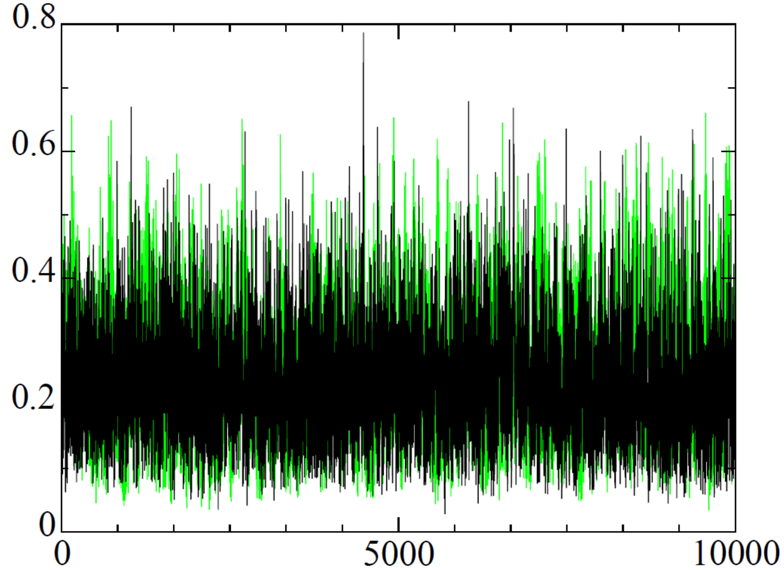
Fast shocks are not sufficient to dissipate energy at the rate it is injected.

Energy accumulates in large-scale coherent structures delimited by intermediate shocks.

For comparison:

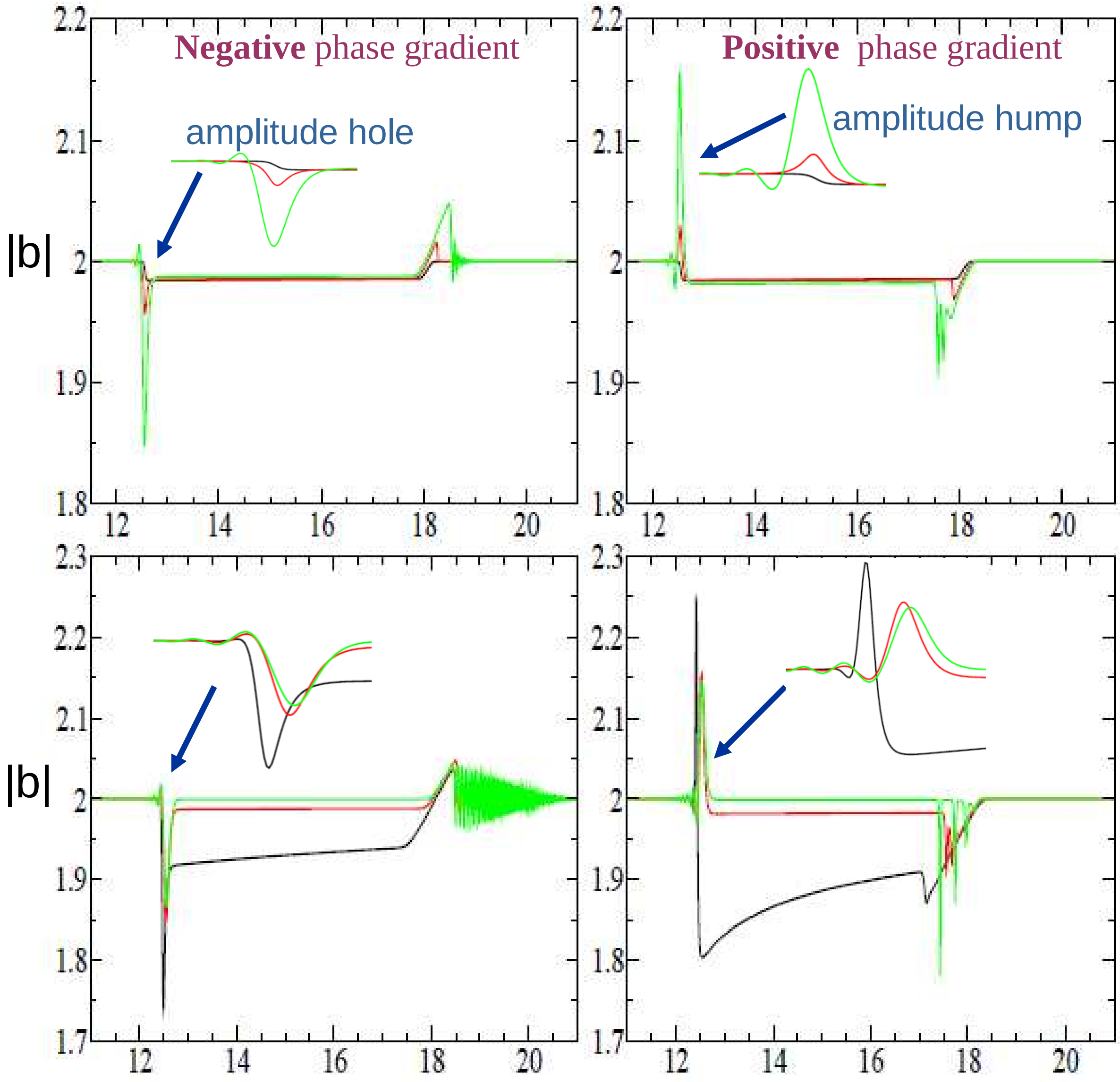
Burgers equation

Well defined limit when $\nu \to 0$.



Influence of dispersion

With dispersion, the **early evolution** of a rotational discontinuity depends on the sign of the phase gradient.



$\eta = 2.10^{-4}$

$\left\{ \begin{array}{l} \delta = 0 \\ \delta = 10^{-3} \\ \delta = 5.10^{-3} \end{array} \right.$

Increasing dispersion increases the overshoot.

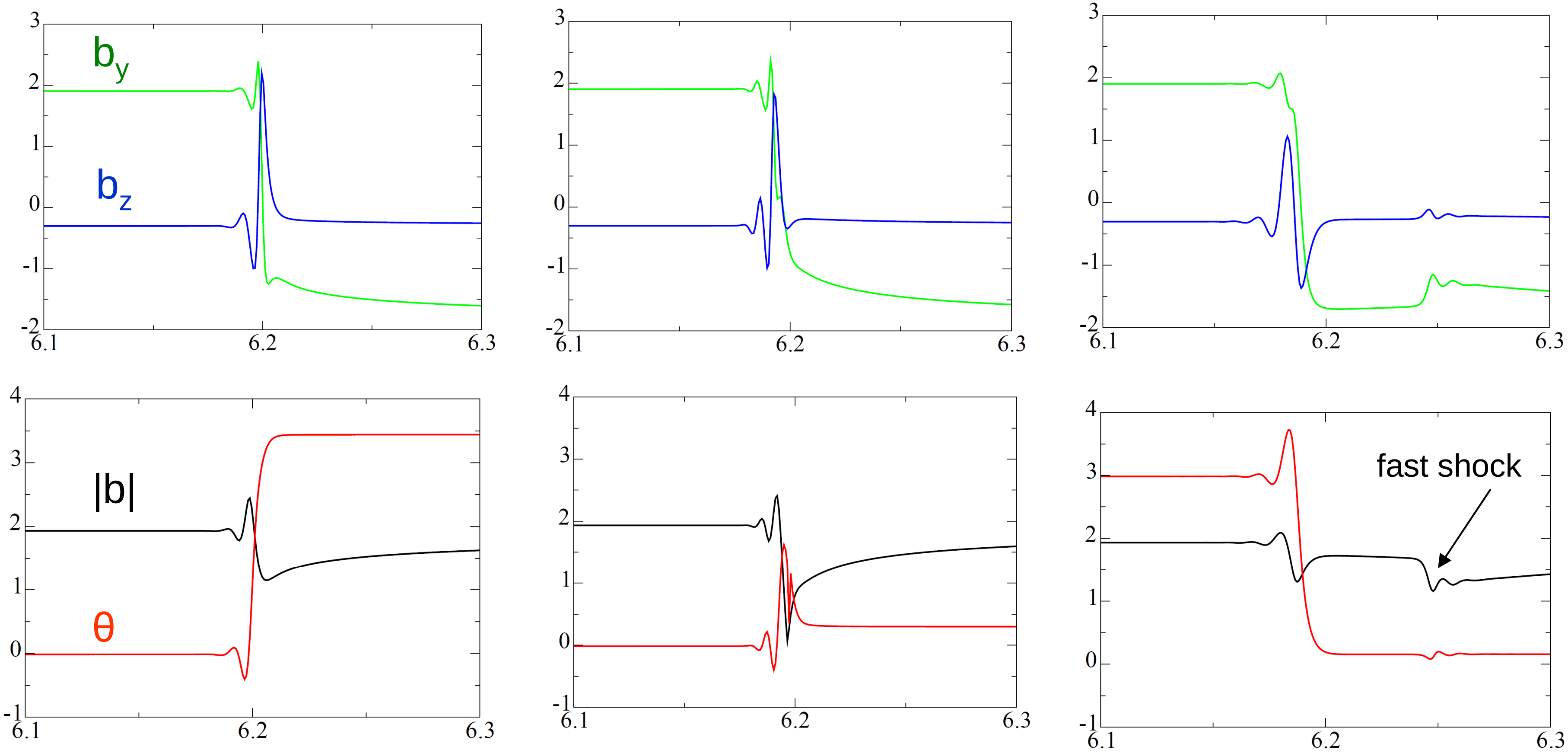
$\delta = 5.10^{-3}$

$\left\{ \begin{array}{l} \eta = 5.10^{-5} \\ \eta = 2.10^{-4} \\ \eta = 10^{-3} \end{array} \right.$

Increasing viscosity increases the strength of the intermediate shock.

"Weak" dispersion: both intermediate and emitted fast shocks are preserved but **overshoots** develop

(phase jump : 1.1 π, $\delta = 10^{-3}$, $\eta = 10^{-4}$)



t = 19.36

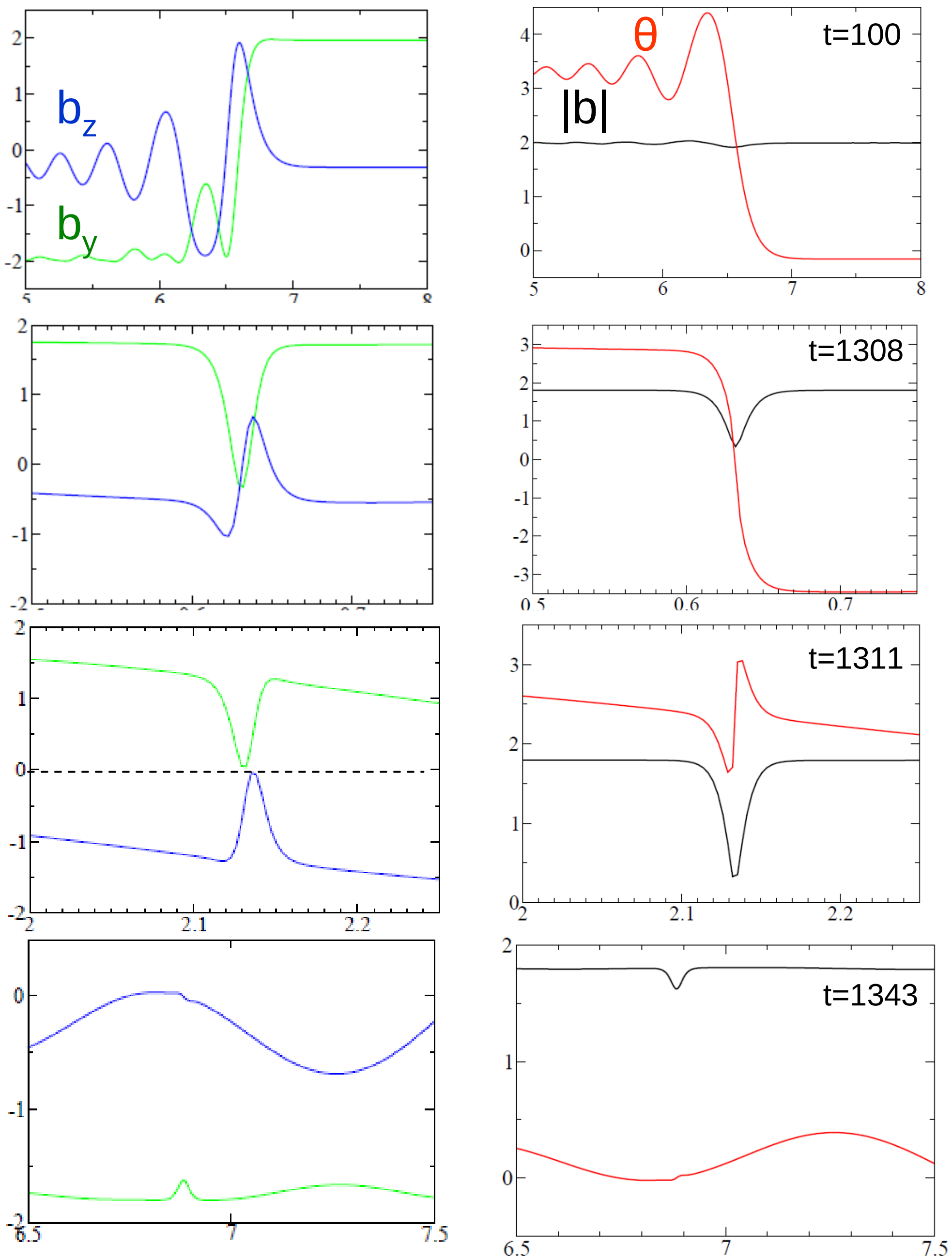
t = 19.38

t = 19.46

Phase reversal
Emission of a fast shock

Similar dynamics when the initial phase jump is reversed.

Larger dispersion: negative phase gradient: ($\Delta\theta=-1.1\ \pi$, $\delta= 5.\ 10^{-3}$, $\eta=10^{-5}$)

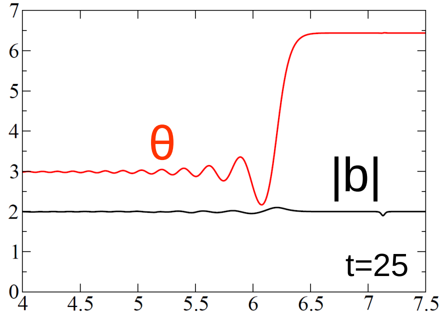


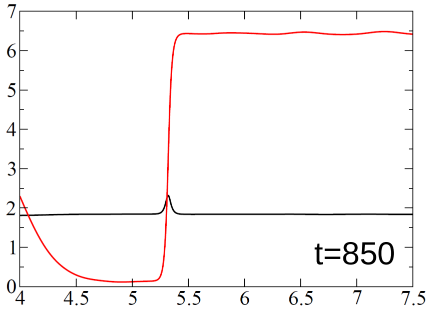
Formation of a **dark soliton**

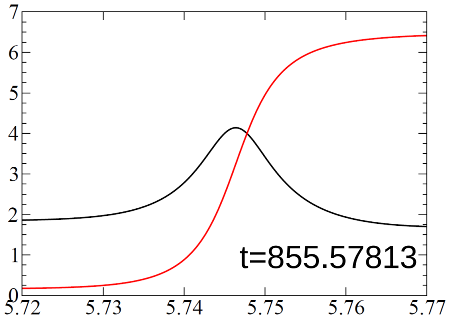
that undergoes **reconnection**

and then rapidly **dissipates (no phase jump)**.

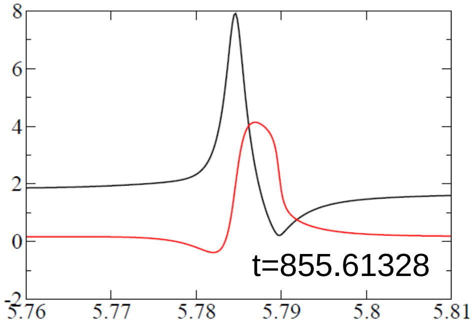
Same dispersion: **positive** phase gradient: ($\Delta\theta$=1.1 π, δ= 5. 10^{-3}, η=10^{-5})

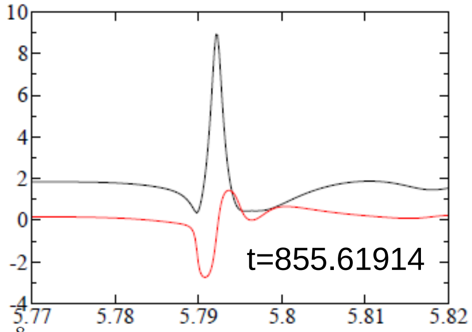


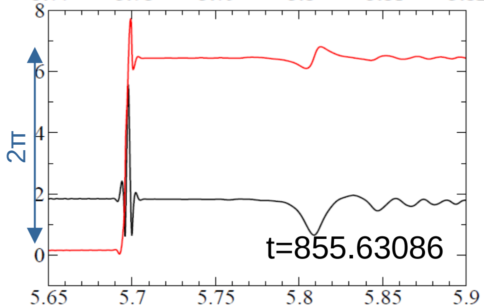




Formation of an amplitude **hump**

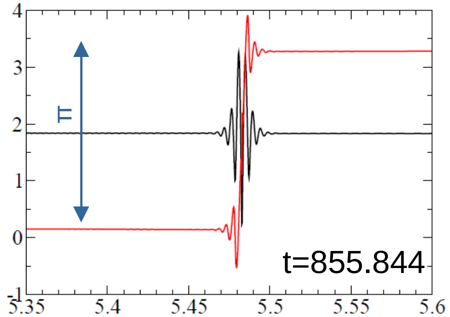


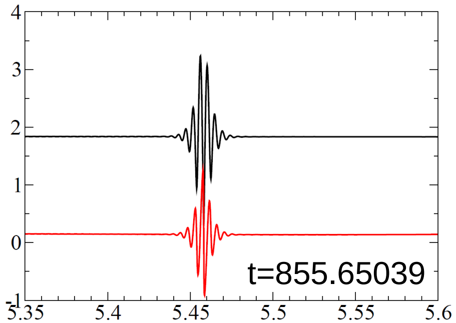




Evolution to an **"oblique" bright soliton** and then to a **breather**.
Strong amplitude growth (**collapse**)

Several phase jumps as time evolves

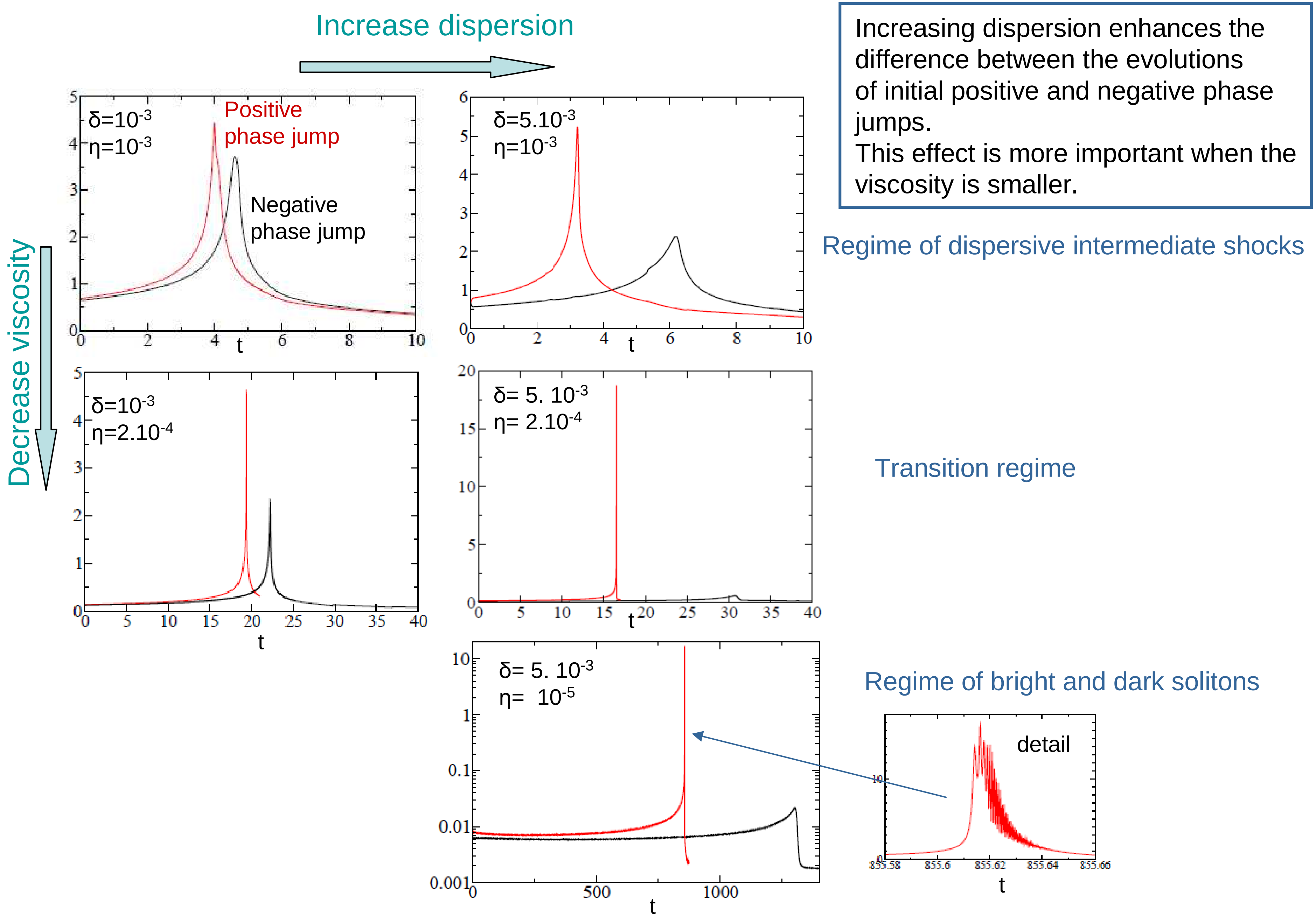




The breather becomes a **wave packet**.
Eventually, **no phase jump**
and **the structure dissipates**.

Time evolution of the dissipation $D(t) = 2\eta \int |\partial_x b|^2 dx$

Increasing dispersion enhances the difference between the evolutions of initial positive and negative phase jumps.
This effect is more important when the viscosity is smaller.



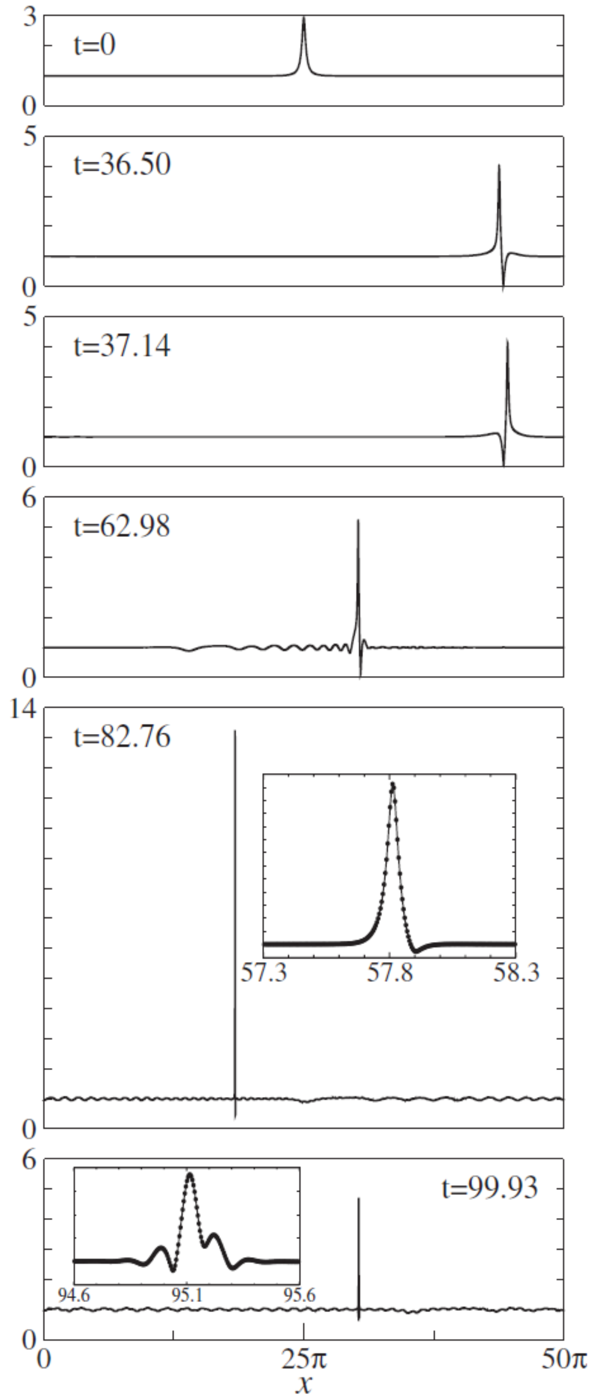
After the initial rotational discontinuity has evolved towards a bright soliton (which displays a 2π phase variation), the later dynamics is similar to that of an initial **bright DNLS soliton with non-zero boundary conditions** (oblique soliton) in the presence of a small dissipation.

This problem was studied in *Sanchez-Arriaga et al., Phys. Rev. E **82**, 016406 (2010)*

Effect of weak dissipation on oblique bright solitons and breathers

Time evolution of an initial bright soliton with λ =0.975 when the DNLS equation with viscosity η=10^{-5}: **QUASI-COLLAPSE.**

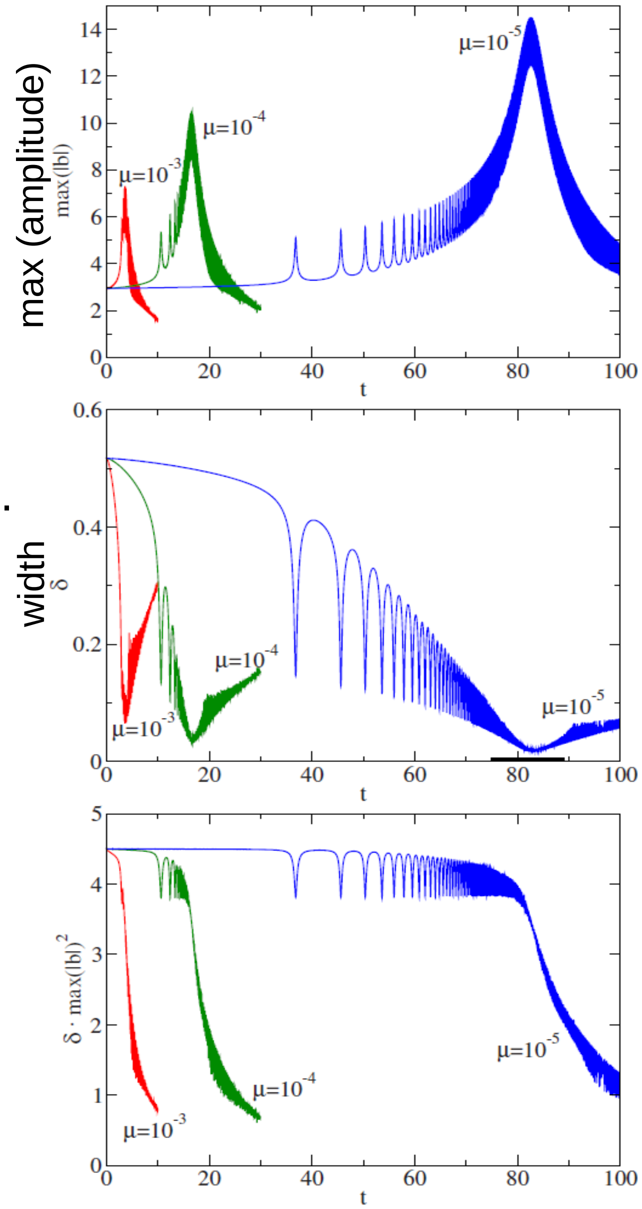
The soliton becomes a breather, collapses, reverses its direction of propagation while radiating, and finally evolves to a wave packet.



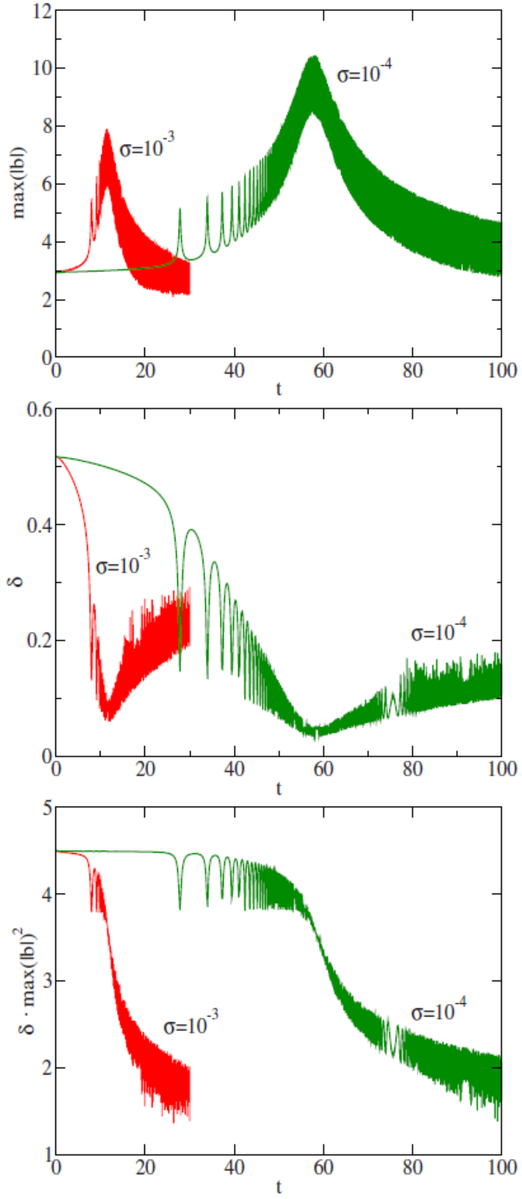
Sanchez-Arriaga et al., Phys. Rev. E ***82****, 016406 (2010)*

$\mu \equiv \eta$

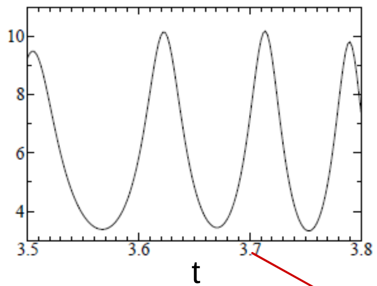
Reducing the viscosity strengthens the amplitude growth and the width reduction, but also delays the process.

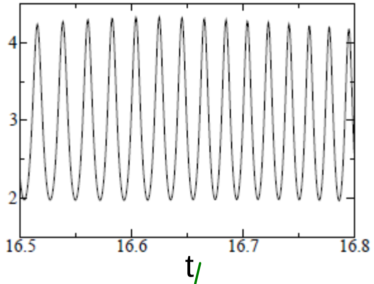


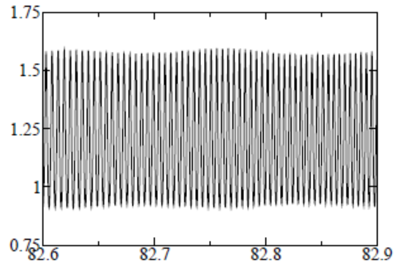
Viscous DNLS



Kinetic DNLS
(dissipation by Landau damping)

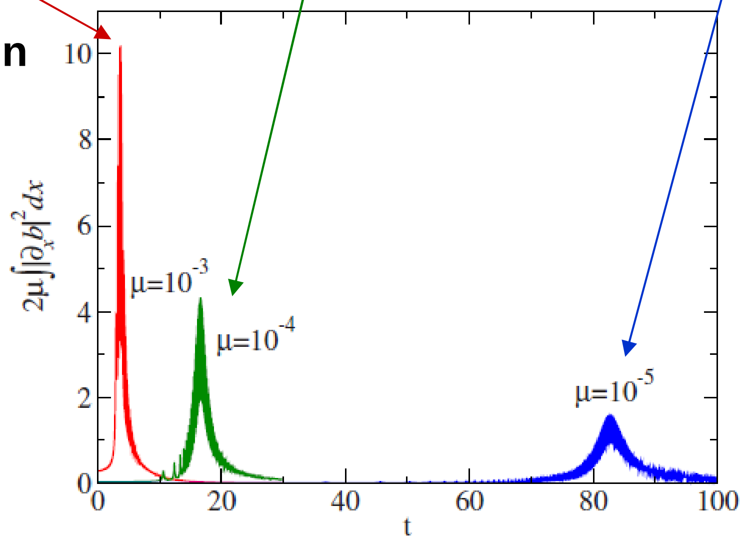




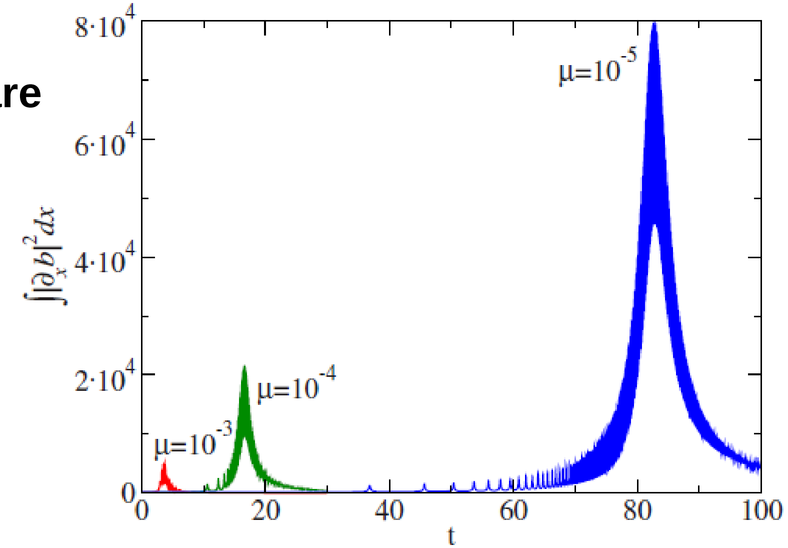


Dissipation

$\mu \equiv \eta$



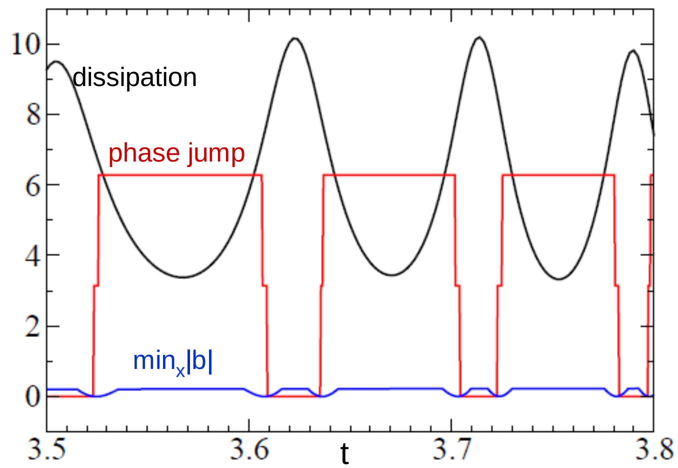
Mean square gradient



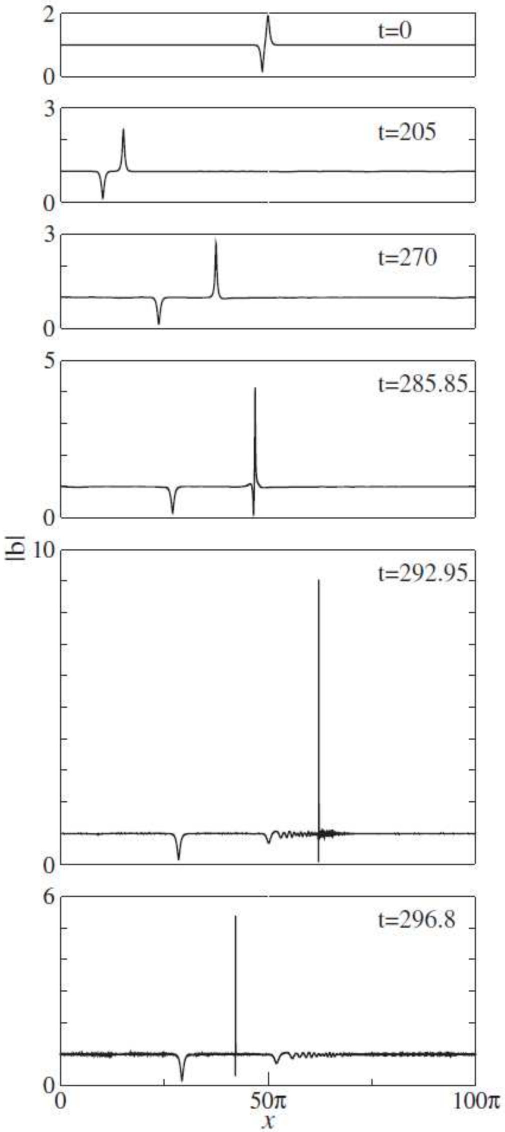
Energy dissipated during the time interval for which D(t) > D_{max}/10, is of the order of 70% of the energy of the initial soliton energy, whatever the viscosity.

As the viscosity is reduced, the global time to dissipate the soliton energy increases, but the oscillation period (which is that of the breather) decreases.

Phase jumps (0 ↔ 2π) at the instants of reconnection ($\min_x|b|$ =0), which also correspond to inflection points of the dissipation (and breather amplitude).



Same phenomenon when starting from a breather rather than from a bright soliton



$\lambda = 0.5 + 0.1i \qquad \mu = 10^{-5}$

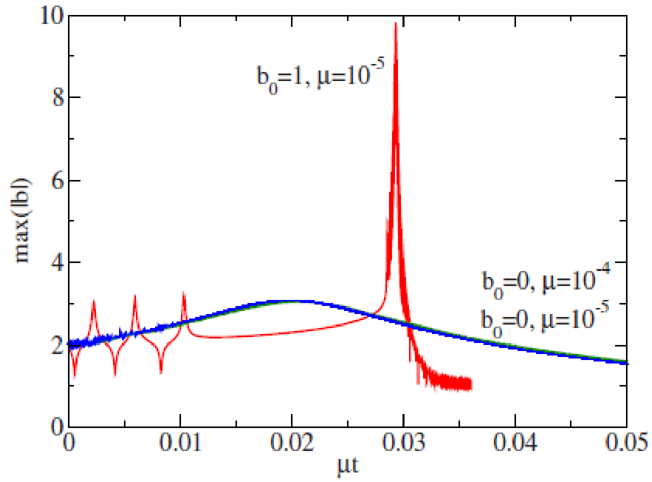


FIG. 10. (Color online) Variation versus μt of the maximal amplitude of the diffusive DNLS equation for initial conditions in the form of initial NVBC soliton with eigenvalue $\lambda = 0.5 + 0.1i$, when $\mu = 10^{-5}$ (red line), and of a VBC soliton with the same eigenvalue, in the cases $\mu = 10^{-5}$ (blue line) and $\mu = 10^{-4}$ (green line). The curves corresponding to the VBC solitons with different μ's superimposed exactly, up to a time rescaling.

Non-zero boundary conditions play a central role in this dynamics, as "oblique solitons" display a 2π phase variation.

Turbulent dispersive regime

A driving term is supplemented to the viscous DNLS equation in the form of a homogeneous random field that approximates a white noise in time and acts at large scales.

Zero initial conditions.

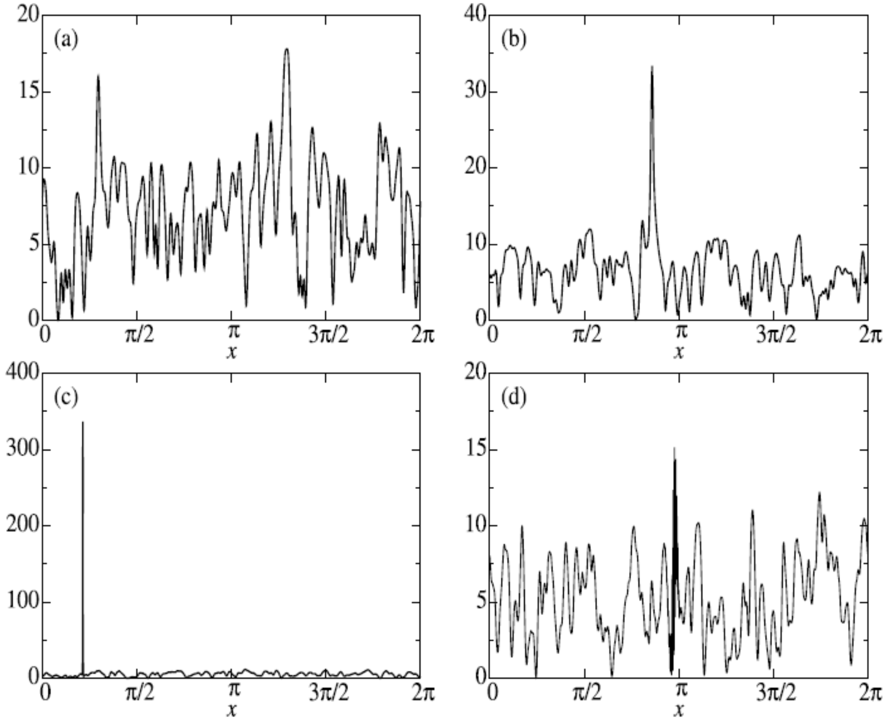


Fig. 1. Intensity $|b|^2$ of the solution of run A at times 1754.0 (a), 1831.0 (b), 1838.7 (c) and 1839.0 (d).

Spontaneous formation of solitons that undergo quasi-collapse.

Laveder et al., Phys. Lett. A ***375****, 3997 (2011)*

Dynamics qualitatively similar to the "turbulent transfer of energy by radiating pulses" *discussed by Rumpf, Newell & Zakharov (PRL 103, 07502, 2009)* :

Direct transfer associated with adiabatic evolution of radiating freak waves whose width decreases.

Distortion of strongly nonlinear structures rather than interaction between mostly linear modes (weak turbulence).

The present model built from an integrable equation cannot develop weak turbulence.

Runs	Dispersion	Diffusion
A	$\delta = 6.25 \times 10^{-2}$	$\mu = 1. \times 10^{-5}$
B	$\delta = 6.25 \times 10^{-2}$	$\mu = 2. \times 10^{-4}$
C	$\delta = 5.00 \times 10^{-3}$	$\mu = 1. \times 10^{-5}$
D	$\delta = 5.00 \times 10^{-3}$	$\mu = 2. \times 10^{-4}$

Time variation of the instantaneous global maxima : rogue wave formation

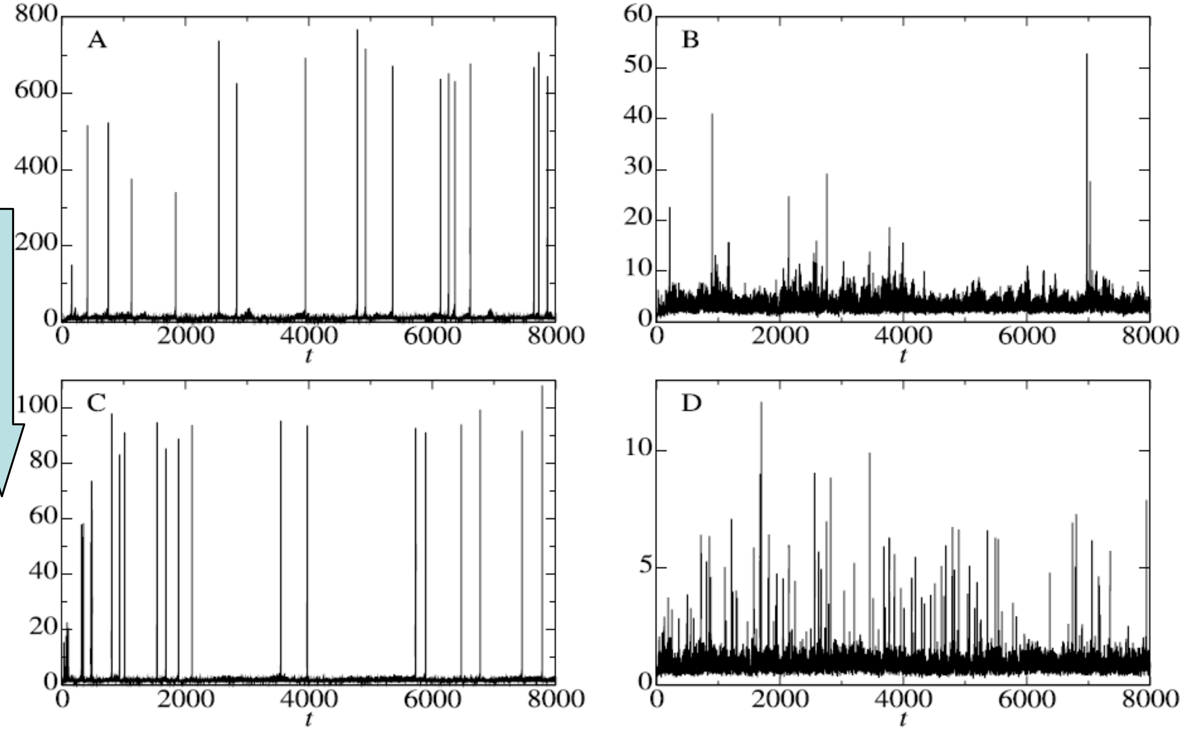
Fixed dispersion but increasing diffusion:
Amplitude of rogue structures is reduced but their frequency enhanced

Increase diffusion

Decrease dispersion

.Reducing dispersion makes the dynamics less intermittent

Life time of the peaks increases with both diffusion and dispersion, ranging from a few hundredths in run C to about a ten in run B.

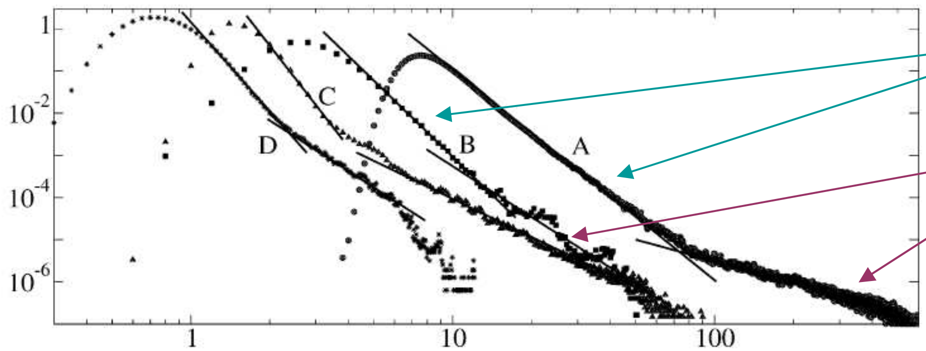


Time evolution of the instantaneous maximum of the AW intensity fluctuations $\sup_x(|b|^2 - \langle|b|^2\rangle)$ for the 4 runs listed in Table 1.

Probability distribution of the instantaneous global maxima

Runs	Dispersion	Diffusion
A	$\delta = 6.25 \times 10^{-2}$	$\mu = 1. \times 10^{-5}$
B	$\delta = 6.25 \times 10^{-2}$	$\mu = 2. \times 10^{-4}$
C	$\delta = 5.00 \times 10^{-3}$	$\mu = 1. \times 10^{-5}$
D	$\delta = 5.00 \times 10^{-3}$	$\mu = 2. \times 10^{-4}$

Two power laws



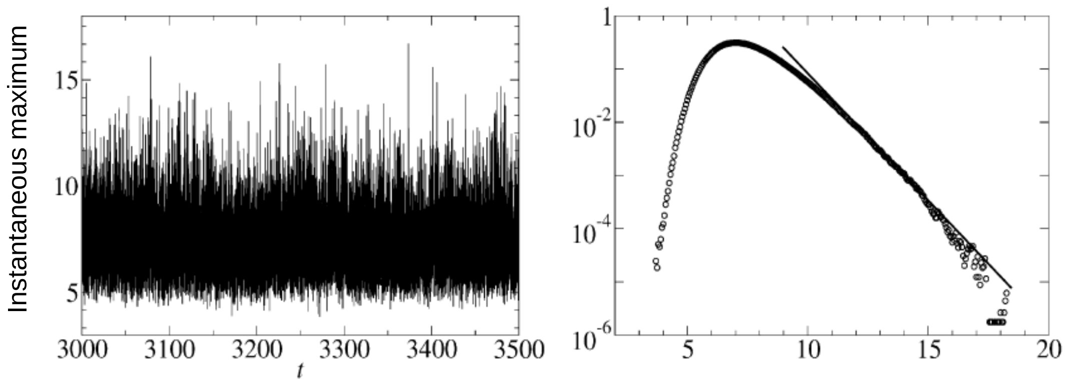
Rogue waves
(large-amplitude solitons)

Very large rogue waves
(collapsing solitons)

Power laws

Fig. 5. Distribution function of the instantaneous maxima of the AW intensity fluctuations $\sup_x(|b|^2 - \langle|b|^2\rangle)$ in log–log scales for runs A (circles), B (squares), C (triangles) and D (asterisks) on the time intervals specified in the text. The straight lines correspond to power laws of exponents −5 and −1.7 (run A), −6 and −4 (run B), −8 and −3 (run C) and −7 and −4 (run D).

Differently, in the integrable case (no dissipation nor forcing)



Exponential decay

Fig. 6. Time evolution of the instantaneous maximum of the AW intensity fluctuations $\sup_x(|b|^2 - \langle|b|^2\rangle)$ (left) and the corresponding distribution function in lin–log scales (right) in the integrable regime that develops in run A, when turning off dissipation and driving at $t = 300$, after the onset of moderate-amplitude solitons but before the formation of rogue waves.

Two power laws in the non integrable regime

Random fluctuations created by the forcing term rapidly evolve to a quasi-solitonic turbulence where structures of different amplitude move chaotically, interact and merge.

(Dynamics made possible by the breaking of the integrability under the action of diffusion and driving).

Some of the formed solitons then strongly amplifiy and collapse.

Two-stage process → two different power laws in the intensity histograms.

Similarities with the two-stage process of rogue-wave formation in water-waves (Zakharov, Dyachenko & Prokofiev, Eur. J. Mech. B, Fluids **25**, 677, 2006).

In both physical situations, during a relatively long period of time, quasi-solitonic turbulence consisting of randomly distributed quasi-solitons with various amplitudes.

In ***water wave*** problem, collapse originates from ***self-focusing*** (a strongly nonlinear process).
Differently, in ***DNLS*** framework, quasi-collapse is due to a ***weak dissipation.***

CONCLUSIONS

Small-amplitude Alfvén waves:

an example of a system possibly subject to *strong topological constraints:*
• intermediate shocks with an angular jump larger than π,
• (oblique) solitons with non-zero boundary conditions (displaying a 2π phase variation).

The solution has to eliminate this topological constraint *through a violent event* (reconnection or quasi-collapse) ***in order to dissipate.***

Impact on the turbulent regime (resulting from a random driving):
Strong temporal intermittency of dissipation characterized by isolated bursts with a power-law probability distribution. In the presence of dispersion, *huge rogue waves*.

Question: Genericity of this phenomenon, described in the context of Cohen-Kulsrud and derivative nonlinear Schrödinger equations?

Instability and collapse of intermediate shocks, similar to CK have been **reproduced in MHD** with anisotropic viscosity and magnetic diffusivity (aimed to mimic Braginski-MHD).

Are there other physical contexts where a similar dynamics can occur?